THE USE OF WEATHER PREDICTIONS AND DYNAMIC PROGRAMMING IN THE CONTROL OF SOLAR DOMESTIC HOT WATER SYSTEMS

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For a solar domestic hot water system, several sophisticated control strategies are presented. Among them are some which use weather forecasts or weather statistics and one which is based on the stochastic version of the dynamic programming algorithm. These control strategies are compared with a simple one by simulations using real weather data of the years 1979 and 1980. The results show reduced energy costs for some of the sophisticated control strategies.

1. Introduction

This study is motivated by the question whether it is possible to save energy if the controller of a solar energy system uses weather predictions or weather statistics, or if the controller is based on the stochastic version of the dynamic programming algorithm [4]. The problem is also of interest for some types of building heating systems [4]. In this paper, the question is studied for a simple solar domestic hot water system.

2. Description of the plant

The plant model consists of a solar collector, a store and an auxiliary electrical heating system (figure 1):

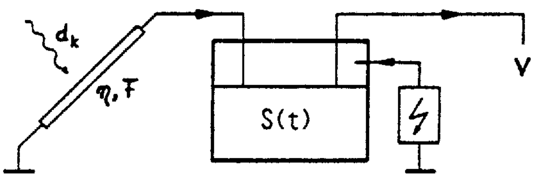

Figure 1 : The plant model

Every day, three phases are distinguished: During the <u>first</u> one, from 6 a.m. to 6 p.m., cold water is heated up in the collector and loaded into the store. During the <u>second</u> phase, between 6 p.m. and 10 p.m., hot water is consumed. We assume that the consumption is constant and equal V. If there is not enough energy in the store during this time, the deficit is compensated by the auxiliary electrical heating system. Supposing this second phase falls into the high rate time, the energy needed is counted μ-fold (μ stand for the ratio of high to low rate). The <u>third</u> and last phase lasts from 10 p.m. to 6 a.m. and coincides with the low rate time. So the store can be preloaded with cheap electrical energy during this phase.

Here, the controller's task of interest consists of deciding at 10 p.m. on how much the store must be preloaded during the following third phase. The purpose is to keep low the costs for electrical energy, or accordingly, the consumption of electrical energy (weighted with μ during the high rate time).

3. Mathematical model of the system

In the mathematical model, the following constants and variables were used.

constants:
- V : consumption during the second phase
- $\xi \cdot V$: storage capacity
- F : collector area
- η : collector efficiency (= const., e.g. heat pipes)

variables (cf. figure 2):
- y_k : storage content at 10 p.m. of day k
- u_k : controller output at day k = value, given by the controller at 10 p.m. of day k, to which the store should be loaded until 6 a.m. of day k+1 ($u_k \in [0, \xi \cdot V]$)

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q_{k+1}: storage content at 6 a.m. of day k+1

d_{k+1}: energy per m² insolated in the collector surface (45°, south) between 6 a.m. and 6 p.m. of day k+1

δ_{k+1}: potential collector output between 6 a.m. and 6 p.m. of day k+1 (phase 1)

r_{k+1}: storage content at 6 p.m. of day k+1 (end of phase 1)

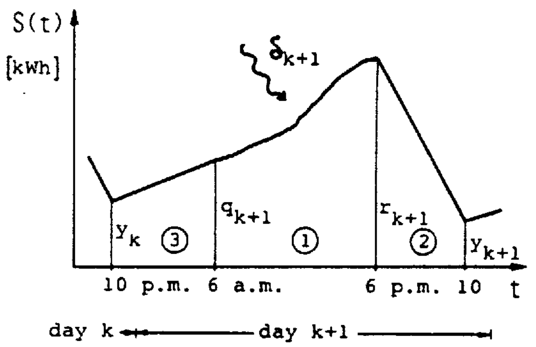

Figure 2 : Storage content S for 24 hours (ⓘ ≙ phase i)

The state variable model of the plant is

$$y_{k+1} = f(y_k, u_k, d_{k+1}) ,$$

where f is described by the following equations.

$$q_{k+1} = \begin{cases} u_k & \text{if } u_k > y_k \\ y_k & \text{if } u_k \leq y_k \end{cases}$$

$$\delta_{k+1} = \eta \cdot F \cdot d_{k+1}$$

$$r_{k+1} = \begin{cases} q_{k+1} + \delta_{k+1} & \text{if } q_{k+1}+\delta_{k+1} \leq \xi \cdot V \\ \xi \cdot V & \text{if } q_{k+1}+\delta_{k+1} > \xi \cdot V \end{cases}$$

$$y_{k+1} = \begin{cases} r_{k+1} - V & \text{if } r_{k+1} - V > 0 \\ 0 & \text{if } r_{k+1} - V \leq 0 \end{cases}$$

The cost of electrical energy during 10 p.m. of day k and 10 p.m. of day k+1 is

$$\ell_k = L(y_k, u_k, d_{k+1})$$
$$= \max(u_k - y_k, 0) + \mu \cdot \max(V - r_{k+1}, 0)$$

where r_{k+1} is given by the equation above. The signal flow diagram of the system is shown in figure 3.

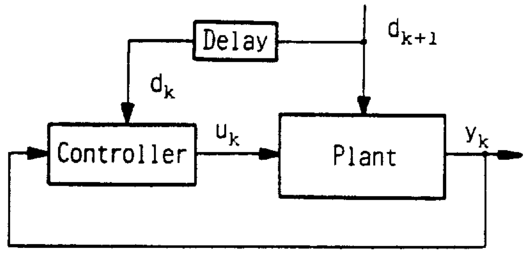

Figure 3 : Signal flow diagram of the system

4. Control strategies

If the controller knew accurately what the weather was like on the following day, an optimum decision would be to calculate on the basis of that knowledge the collector output of the next day. Thus, the store would be preloaded with cheap electrical energy of the third phase up to that amount of energy which, together with the calculated collector output, yields just the right amount to cover the consumption at the following evening.
In this case, no expensive electrical energy is needed during the consumption phase. However, this ideal control strategy cannot be realized, because the controller does not know in advance the solar energy input.

However, the controller can use a weather prediction and deal with it in the above described way. Then it is a so-called "certainty-equivalence controller". In the following, three such certainty-equivalence controllers are outlined. Common to them is

$$u_k = V - \eta \cdot F \cdot \hat{d}_{k+1}$$

where $\hat{d}_{k+1}$ is the prediction of d_{k+1} made at 10 p.m. of day k.

Strategy 1: Certainty-equivalence controller with official weather forecasts.

The official weather forecasts of the Swiss Meteorological Institute were used. As described in [2] or [6], these weather forecasts are characterized by p which can assume the following values:

"Weather forecast" p_{k+1}	0	1	2
$\hat{d}_{k+1} = d_{pred}(p_{k+1})$ [kWh/m²d]	0.5	1.5	3.5

Table 1 : Assignment of $\hat{d}_{k+1}$ to the forecast p_{k+1}

The predicted collector output of the following day is:

$$\hat{c}_{k+1} = \eta \cdot F \cdot d_{pred}(p_{k+1})$$

and the control law is:

$$u_k = V - \eta \cdot F \cdot d_{pred}(p_{k+1})$$

Strategy 2: Certainty-equivalence controller with the prediction "tomorrow equal today".

At 10 p.m. the controller predicts for the following day a collector output which is equal to the collector output of the current day. It is

$$\hat{d}_{k+1} = d_k$$

$$u_k = V - \eta \cdot F \cdot d_k$$

Strategy 3: Certainty-equivalence controller with a conditional expected value as a prediction.

Every day at 10 p.m., the controller determines the conditional expected value of the collector output of the following day, given the output of the current day. The irradiated energy d_k on the collector surface (45°, south) is divided into 4 classes and a typical value of d_k is assigned to each class.

d_k [kWh/m²d]	0–2	2–4	4–6	6–8
class number $\gamma_k = \gamma(d_k)$	1	2	3	4
typ. value for $d_k = d^*(\gamma_k)$	0	2	4	6

Table 2 : Assignment of $\gamma(d_k)$ to d_k and $d^*(\gamma_k)$ to γ_k.

The prediction of d_{k+1} is:

$$\hat{d}_{k+1} = E[d^*(\gamma_{k+1}) | \gamma_k = \gamma(d_k)]$$

$$= \sum_{j=1}^{4} d^*(j) \cdot P_r(\gamma_{k+1} = j | \gamma_k = \gamma(d_k))$$

where $P_r(\gamma_{k+1} = j | \gamma_k = i)$ is the transition probability from state i to state j. It is determined by measured weather data from 1963 to 1972 and is different for each month [6]. E.g. for April

i\j	1	2	3	4
1	0.37	0.29	0.25	0.09
2	0.29	0.32	0.25	0.14
3	0.24	0.29	0.27	0.20
4	0.14	0.14	0.23	0.50

Table 3 : Transition probabilities $P_r(\gamma_{k+1}=j|\gamma_k=i)$ (for April and Zürich-Kloten)

The control law is:

$$u_k = V - \eta \cdot F \cdot \sum_{j=1}^{4} d^*(j) \cdot P_r(\gamma_{k+1}=j|\gamma_k=\gamma(d_k))$$

In the following, there are two strategies presented which are not in the class of the certainty-equivalence controller.

Strategy 4: Minimizing the conditional expected value of the electrical energy over a day.

For different values u_k, up to which the store can be preloaded in the following low rate phase, the controller calculates the conditional expected value of the cost ℓ_k for electrical energy during the next 24 hours, given the collector output of the current day.
That value of u_k is selected which minimizes

$$E[\ell_k | \gamma_k = \gamma(d_k)]$$

$$\text{where } \ell_k = L(y_k, u_k, d^*(\gamma_{k+1})).$$

Therefore,

$$u_k \text{ minimizes } \sum_{j=1}^{4} L(y_k, u_k, d^*(j)) \cdot P_r(\gamma_{k+1}=j|\gamma_k=\gamma(d_k))$$

where L is given by equations of section 3 and $P_r(\gamma_{k+1}=j|\gamma_k=i)$ by table 3.

Strategy 5: Control strategy optimized by the dynamic programming algorithm.

Here, the expected value of the cost for electrical energy over a month is minimized. The plant model is expanded by a stochastic model for d_k.

$d_k = d^*(\gamma_k)$

γ_k is a Markov chain with the transition probabilities given in [6] (for April see table 3).

Now the expanded system can be regarded as a system with the state vector

$$\underline{x}_k = \begin{bmatrix} y_k \\ \gamma_k \end{bmatrix}$$

To this system the optimum control strategy

$$u_k = g_k^*(y_k,\gamma_k) \qquad k = 1,2,\dots,M-1$$

which minimizes the expected value of the cost over a month

$$E\left[\sum_{k=1}^{M-1} \ell_k\right]$$

is to be found. This can be done by stochastic version of the dynamic programming algorithm (e.g. in [5]).
The dynamic programming algorithm applied to the expanded system model gives the following algorithm which allows to calculate the optimum control law $g_k^*(y_k,\gamma_k)$ for $k = 1,2,\dots,M-1$.

$$J_M^*(y_M,\gamma_M) = 0$$

For $k = M-1, M-2, \dots, 2, 1$ calculate the functions

$$g_k^* : \quad (y_k,\gamma_k) \longmapsto g_k^*(y_k,\gamma_k)$$

$$J_k^* : \quad (y_k,\gamma_k) \longmapsto J_k^*(y_k,\gamma_k)$$

where $g_k^*(y_k,i)$ is the value of $u_k \epsilon [0,\xi\cdot V]$ which minimizes

$$J_k(y_k,i,u_k) = \sum_{j=1}^{4} (L\{y_k,u_k,d^*(j)\}$$

$$+ J_{k+1}^*\{f(y_k,u_k,d^*(j)),j\})\cdot P_r(\gamma_{k+1}=j|\gamma_k=i)$$

$$J_k^*(y_k,\gamma_k) = J_k(y_k,\gamma_k,g_k^*(y_k,\gamma_k))$$

$L(\dots)$ and $f(\dots)$ are given in section 3 and $P_r(\gamma_{k+1}=j|\gamma_k=i)$ by table 3.

For the numerical calculations, y_k is quantizied with a quantization step of 1 kWh.

This algorithm is the time variant version of the dynamic programming algorithm. For the simulations, the time invariant version is used.

$$u_k = g^*(y_k,\gamma_k) = \lim_{k\to-\infty} g_k^*(y_k,\gamma_k)$$

In the following chapter, there are two tables with results. For a comparison, on the first line of the tables the results are given for a simple strategy (strat.0).

Strategy 0:

There, the store is preloaded during the night, always on the same constant value. This value was optimized with real weather data of the period 1963 to 1972 for a store with the size $\xi\cdot V = 1.5$ V and $F = 10$ m^2.

5. Results

The strategies until no. 5 are based on probability distributions which are determined by measured weather data from 1963 to 1972. The simulations are performed with weather data from 1979 and 1980. It is emphasized that the data used in simulations are different from a priori data on which the control strategies are based.

For the presented results the parameters were:

consumption $V = 21.77$ kWh/d
storage capacity $= \xi\cdot V$
collector: $F = 10$ m^2, 45° inclination, south orientation, $\eta = 50\%$
location of the plant: Zürich-Kloten
weather data and forecasts are from [1]

Strategy	April 79+80	July 79+80	Oct. 79+80
0) equal every day	499.3	417.7	878.4
1) weather forecasts	576.5	461.2	822.3
2) tomorrow = today	503.2	320.8	878.7
3) cond.expected val.	469.3	331.2	808.3
4) min. over a day	485.1	339.6	845.6
5) dyn. programming	515.5	330.5	873.9
performance bound	350.8	195.5	670.3

Table 4 : Weighted electrical energy for $\xi = 1.3$

One can see in table 4 that for $\xi = 1.3$ the two strategies No. 3 and No. 4, using weather statistics are better than strategy 0. In the last line, there is the lowest bound of the cost, which results from the unrealistic assumption that accurate predictions $\hat{d}_{k+1}$ (= d_{k+1}) are available. The results of the strategies are quite far away from the performance bound.

For the next table, only the storage size is changed. Instead of 1.3, ξ is 2. In this case, strategy 1 is always better than strategy 0.

Strategy	April 79+80	July 79+80	Oct. 79+80
0) equal every day	381.5	233.9	843.7
1) weather forecasts	362.7	225.1	709.7
2) tomorrow = today	406.4	255.6	795.0
3) cond.expected val.	390.2	278.7	730.0
4) min. over a day	372.9	283.5	716.5
5) dyn. programming	400.7	242.5	688.7
performance bound	327.4	177.2	670.3

Table 5 : Weighted electrical energy for $\xi = 2$

In [6] some more strategies are tested, and the influence of the size of the storage on the costs is studied.

6. Conclusion

For a simple solar domestic hot water system, several sophisticated control strategies were presented. The design was based on a priori weather data from 1963 to 1972. In a simulation study with weather data of the years 1979 and 1980 they were compared with a simple optimized strategy. Lower energy costs could be achieved with some of the sophisticated strategies. The ranking list is dependent on the plant parameters as ξ.

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