Model Predictive Control of Building Heating System

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SUMMARY

Conventional control strategies of a building heating system such as weather-compensated heating control cannot make use of the energy supplied to a building (e.g. solar gain in case of sunny day). On the other hand, dropout of outside temperature can lead to underheating of a building. Model predictive controller presented in the article combines weather forecast and thermal model of a building and predicts trends of inside temperature. By this, it can utilize thermal capacity of a building and minimize energy consumption. It can also maintain inside temperature at desired level independent of outside weather conditions. The controller was tested on a real building and results were compared with the present heating control.

1 INTRODUCTION

According to the U. S. Energy Information Administration, in 2005, buildings accounted for 39 % of total energy usage, 12 % of the total water consumption, 68 % of total electricity consumption, and 38 % of the carbon dioxide emissions in the U. S. A. [1]. Although the energy efficiency of systems and components for heating, ventilating, and air conditioning (HVAC) has improved considerably over recent years, there is still potential for substantial improvements. This article deals with an advanced control technique, that can provide significant energy savings in comparison with conventional, non-predictive techniques.

Widely used control strategy of water heating systems is the weather-compensated control. This feedforward control can lead to poor energy management or reduced thermal comfort even if properly set up, because it utilizes current outside temperatures only. Weather conditions, however, can change dramatically in few hours; and due to the heat accumulation in large buildings, it can lead to underheating or overheating of the building easily.

During recent years, significant advances have been done for the HVAC control systems. For instance, continuous adaptation of control parameters, optimal start-stop algorithms, or inclusion of free heat gains in the control algorithm are particular improvements of the building heating system. The model predictive controller presented in this article introduces a different approach to the heating system control design. As the outside temperature is one of the most influential quantity for the building heating system, weather forecast is employed in the predictive controller. It enables to predict inside temperature trends according to the selected control strategy. The aims of the control can be expressed in natural form as thermal comfort and economy trade off.

Figure 1: The building of the Czech Technical University in Prague that was used for MPC application

2 MODEL PREDICTIVE CONTROL

Model (Based) Predictive Control (MPC) is a method of advanced control originated in late seventies and early eighties in the process industries (oil refineries, chemical plants, ...) ([2, 3, 4, 5]). The MPC is not a single strategy, but a vast class of control methods with the model of the process explicitly expressed trying to obtain control signal by minimizing objective function subject to (in general) some constraints.

The minimization is performed in an iterative manner on some finite optimization horizon to acquire N step ahead-prediction of a control signal that leads to the minimum value of the criterion, subject to all constraints. This, however, carries lots of drawbacks such as no feed-back, no robustness, no stability guarantee, etc. Many of these drawbacks can be overcome by applying so-called receding horizon, i.e. at each iteration only the first step of the control strategy is implemented and the control signal is calculated again, thus, in fact, the prediction horizon keeps being shifted forward. Stability of the constrained receding horizon has been discussed in [6, 7], or yet another approach using robust control design approach in [8].

There were several attempts made to utilize predictive control concept in HVAC in the last decade [9, 10, 11]. Complex view into area of optimal building control gives the project OptiControl[1]. Besides its own results, it also provides a wide range of references to the related articles. Another project worth to mention is the Predictive Networked Building Control that deals with predictive control of the thermal energy storage on the campus of the UC-Berkeley[2]. Most of the articles devoted to the HVAC predictive control conclude results just by numerical simulations. On the contrary, this article describes MPC being tested on the ***real eight-floor building*** (see Fig. 1).

2.1 Model identification

One of the crucial contributors to the quality of the control is a well identified model which will be later on used for control in MPC algorithm. There are several completely different approaches to system identification including physical modeling, CFD modeling or statistical identification. As traditional methods are rather time consuming for buildings, we have turned towards statistical identification methods, and, more specifically, towards subspace methods [12, 13, 14].

The objective of the subspace algorithm is to find a linear, time invariant, discrete time

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[1]http://www.opticontrol.ethz.ch
[2]http://sites.google.com/site/mpclaboratory/research/
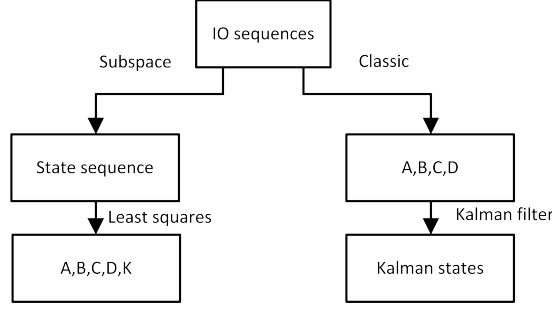   predictive-networked-building-control-1

Figure 2: Comparison between classical and subspace identification methods

model in an innovative form

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) + Ke(k) \\
y(k) &= Cx(k) + Du(k) + e(k),
\end{aligned}
\tag{1}
$$

where A, B, C, and D are system matrices, K is Kalman gain – derived from the Algebraic Riccati Equation (ARE) [15], and e is a white noise sequence. This model is equivalent to the well-known stochastic model

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) + w(k) \\
y(k) &= Cx(k) + Du(k) + v(k),
\end{aligned}
\tag{2}
$$

with

$$
\operatorname{cov}(w,v) = E\left(\begin{bmatrix} w(p) \\ v(p) \end{bmatrix} \begin{bmatrix} w^T(q) & v^T(q) \end{bmatrix}\right) = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \delta_{pq} \geq 0,
\tag{3}
$$

wherein matrices Q, S and R are covariance matrices of process and measurement noise sequences, respectively. Loosely speaking, the objective of the algorithm is to determine the system order n and to find the matrices A, B, C, D and K.

The main difference between classical and subspace identification is, given the sequence of input data $u(k)$ and output data $y(k)$, as follows:

- **Classical approach.** Find system matrices, then estimate the system states, which often leads to high order models that have to be reduced thereafter.
- **Subspace approach.** Use orthogonal and oblique projections to find Kalman state sequence (see [15]), then obtain the system matrices using least squares method.

The differences in the approaches can be seen in Fig. 2[16]. The entry point to the algorithm are input-output equations as follows:

$$
\begin{aligned}
Y_p &= \Gamma_i X_p^d + H_i^d U_p + Y_p^s \\
Y_f &= \Gamma_i X_f^d + H_i^d U_f + Y_f^s \\
X_f^d &= A^i X_p^d + \Delta_i^d U_p,
\end{aligned}
\tag{4}
$$

where Y_p and Y_f are the Hankel matrices of past and future outputs, U_p and U_f are the Hankel matrices of past and future inputs, X_p^d and X_f^d are the deterministic Kalman state sequences, Y_p^s and Y_f^s are the stochastic Hankel matrices of past and future outputs, H_i^d is the lower block triangular Toeplitz matrix for the deterministic subsystem (which contains all matrices A, B, C and D), Γ_i is the extended system observability matrix (which contains the system matrices

A and *C*) and Δ_i^d is the deterministic reversed extended controllability matrix (which contains system matrices *A* and *B*). More details about constructing said matrices can be found in [12, 16, 17]. Using combined algorithm described in [17], we get

$$\begin{bmatrix} \widetilde{X}_{i+1} \\ Y_{i|i} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \widetilde{X}_i \\ U_{i|i} \end{bmatrix} + \begin{bmatrix} \rho_w \\ \rho_v \end{bmatrix}, \tag{5}$$

with $\widetilde{X}_i = \widetilde{X}_{i_{\widehat{X}_0,P_0}}$, where $\widehat{X}_0$ is oblique projection defined by [17] as:

$$\widehat{X}_0 = X_p^d /_{U_f} U_p, \tag{6}$$

where P_0 is state covariance matrix, and we can determine noise sequence covariance matrices *Q*, *S* and *R* from the residuals, which are defined by Eq. (3). Solving Eq. (5) using least squares methods, we get the state space system description of the system, namely the system in the innovation form (Eq. (1)) with matrices *A*, *B*, *C*, *D* and *K*.

2.2 Predictive controller

MPC strategy. The MPC strategy comprises two basic steps:
- The future outputs are predicted in an open-loop manner using the model provided information about past inputs, outputs and future signals, which are about to be calculated. The future control signals are calculated by optimizing the objective function, i.e. chosen criterion, which is usually in the form of quadratic function. The criterion constituents can be as follows:
    - errors between the predicted signal and the reference trajectory $r(k)$
    - control effort
    - rate of change in control signals
- The first component of the control sequence $u(k)$ is sent to the system, whilst the rest of the sequence is disposed. At the next time instant, new output $y(k+1)$ is measured and the control sequence is recalculated, first component $u(k+1)$ is applied to the system and the rest is disposed. This principle is repeated continuously (receding horizon).

The reference trajectory $r(k)$, temperature in the room in our case, is known in advance as a schedule. The major advantage of MPC is the ability of computing the outputs $y(k)$ and corresponding input signals $u(k)$ in advance, that is, it is possible to avoid sudden changes in control signal and avoid the undesired effect of delay in system response.

MPC problem formulation. For given linear, time invariant, discrete model

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + Du(k) \tag{7}$$

find the optimal control sequence on the horizon of prediction (length *T*) by minimizing the objective function

$$J = \sum_{k=0}^{T-1} q(t)\,(y(k) - r)^2 + r(k)u(k)^2, \tag{8}$$

subject to

$$\begin{aligned} u_{min} \leq \quad & u \quad \leq u_{max} \\ (y-r)_{min} \leq \quad & (y-r) \quad \leq (y-r)_{max} \\ \Delta_{min}\xi \leq \quad & \Delta\xi \quad \leq \Delta_{max}\xi \end{aligned} \tag{9}$$

where constraints u_{min}, $(y-r)_{min}$, $\Delta\xi_{min}$ and u_{max}, $(y-r)_{max}$, $\Delta\xi_{max}$ are minimum and maximum values of the control signal, error of the output signal from reference and a rate of change of

control signal or error of the output signal from reference, respectively. J is the value of the objective function, r is the reference trajectory, y and u are output and control signal (denoted without specification of a time instant k). The criterion (Eq. (8)) can be rewritten into a matrix form

$$J = (y - r)^T Q(y - r) + u^T Ru, \tag{10}$$

where Q and R are weighting matrices of output error and control effort, respectively. The trajectory of the output is given as:

$$
\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(T-1) \end{bmatrix} =
\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{T-1} \end{bmatrix} x_0 +
\begin{bmatrix} D & & & \\ CB & D & & \\ \vdots & & \ddots & \\ CA^{T-2}B & \dots & CB & D \end{bmatrix}
\begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(T-1) \end{bmatrix}, \tag{11}
$$

i.e.

$$y = \Gamma x_0 + Hu, \tag{12}$$

where Γ is observability matrix and H is matrix of impulse responses. Using Eq. (12), we can rewrite Eq. (10) as follows:

$$J = (\Gamma x_0 + Hu - r)^T Q(\Gamma x_0 + Hu - r) + u^T Ru. \tag{13}$$

If the constraints (Eq. (9)) are not taken into account, model of the system is linear, and the criterion is quadratic, minimization problem Eq. (8) can be solved analytically – using methods such as completing the square or Lagrange multipliers, etc., leading to the optimal control sequence in the form of

$$u = -(H^T QH + R)^{-1} H^T Q(\Gamma x_0 + Hu - r), \tag{14}$$

otherwise iterative methods have to be used. In the case of quadratic criterion and constrained optimization problem, a quadratic programming is one of the most popular solutions built in the most of the modern optimization packages. Alongside to the many commercial software, such as APC Library (Siemens), Pavilion8 (Pavilion Technologies), ADMC & DMCX1 (Cutlertech), RMP(Honeywell), etc., there is also a free software, e.g. Scilab[3] and its optimization routines.

3 CASE STUDY

The methods described in the previous section were tested in spring 2009 at the building of the Czech Technical University in Prague. All algorithms were implemented in Scilab.

3.1 Description of the Building

The building of the Czech Technical University in Prague uses a "Crittall" [18] type ceiling radiant heating and cooling system. The "Crittall" system, invented in 1927 by R. G. Crittall and J. L. Musgrave, was a favorite heating system in the Czech Republic during 1960's for large buildings. In this system, the heating (or cooling) beams are embedded into the concrete ceiling. The control of individual rooms is very complicated due to the technical state of the control elements in all rooms. The control is therefore carried out for one entire building block, i.e. the same control effort is applied to all rooms of the building block. There are three building blocks with the same construction and orientation. Therefore, this situation is ideal for comparison of different control strategies, as depicted in Fig. 4.

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[3]Open source scientific software package for numerical computations (http://www.scilab.org/)
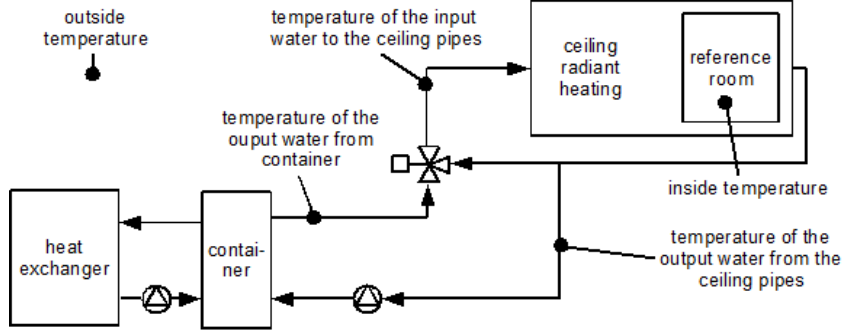
Figure 3: Simplified scheme of the ceiling radiant heating system.

A simplified scheme of the ceiling radiant heating system is illustrated in Fig. 3. The source of heat is a vapor-liquid heat exchanger, which supplies the heating water to the water container. A mixing occurs here, and the water is supplied to the respective heating circuits. An accurate temperature control of the heating water for respective circuits is achieved by a three-port valve with a servo drive. The heating water is then supplied to the respective ceiling beams. There is one measurement point in a reference room for every circuit. The setpoint of the control valve is therefore the control variable for the ceiling radiant heating system in each circuit.

3.2 Description of the model

The ceiling radiant heating system was simplified into a linear, time invariant mathematical model. Outside temperature prediction and heating water temperature were used as the model inputs. Prediction of the outside temperature is composed of two values, T_{max} and T_{min}, defining a confidence interval. The only output of the model was the inside temperature. This can be formalized according to Eq. (7) as

$$x(k+1) = Ax + B \begin{bmatrix} T_{min} \\ T_{max} \\ T_{hw} \end{bmatrix}, T_{in} = Cx + D \begin{bmatrix} T_{min} \\ T_{max} \\ T_{hw} \end{bmatrix}, \tag{15}$$

where T_{hw} is the temperature of the heating water and T_{in} denotes the inside temperature. The state x has no physical interpretation, when identified by means of the subspace identification. System order is determined by the identification algorithm.

Modeling of the heating system of the CTU building is discussed in detail in [19].

3.3 Results

Two nearly identical blocks of the CTU building were used for testing. The first block was controlled by the weather-compensated controller, while the second one by predictive controller. The current weather compensated control is well tuned up thanks to long term experience with the building. Testing was performed from March 24 to March 30, 2009, which was the end of the heating season in the Czech republic. That highlights the advantages of the predictive control, because there are days when there is no need of heating at all. The model predictive controller was able to identify those gaps where no control was needed, thus save energy. This can be seen in Fig. 4.

The efficiency of the predictive control was superior to the weather-compensated controller.

4 REMARKS TO FUTURE DEVELOPMENT

As already stated, subspace identification represents one of the black-box approaches to system modeling. This, alongside with its advantages, carries also some drawbacks:
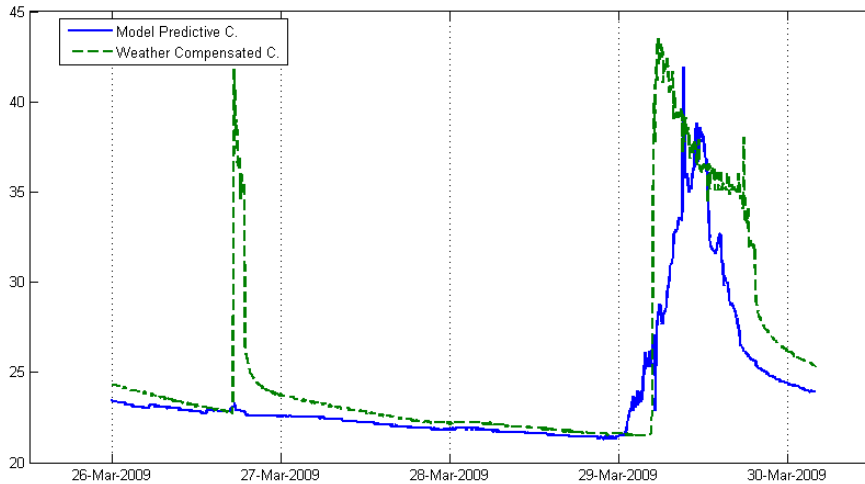
Figure 4: Measured trends of the heating water temperature controlled by the weather compensated control and the predictive controller

- The system might not be excited enough [12], i.e. the input of the system does not excite the system on satisfactory number of frequencies, thus identification algorithms lack considerable amount of information.
- User may have knowledge of some key feature or characteristics of the physical essence of the system, which is "lost" in the number of data.
- Natural characteristics of the data might pose considerable statistical problem.

Future development of the identification algorithm will try to remedy the above-mentioned problems. One way which is still not full discovered[4] is incorporating of prior information. This approach make use of Bayesian approach [20] to the system identification and changes nature of the subspace identification method towards grey-box identification.

Another aspect of the identification is the persistency of the excitation or the excitation itself. Data gathered from the measurement lack some important physical characteristics of the building. One of the possible approaches how to deal with this weak point is to generate artificial data that already contain the desired properties. There is also another possibility, more expensive though – a specially proposed experiment on real building, which is about to be done in the beginning of 2010.

5 ACKNOWLEDGMENT

6 CONCLUSION

It is obvious, that predictive control has a great potential in the area of building heating control. Especially in case of buildings with great heat accumulation capabilities. The results from one week testing in spring 2009 are very encouraging. Testing confirmed our empirical

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[4]`Some methods (SWLRA) algorithms are capable of handling incorporation of prior information to SISO system, but algorithm for MIMO system is still missing`

experiences and the efficiency of the predictive controller was superior to the present well tuned weather compensated controller. However, it is a complicated technique and launching of the predictive controller requires deep knowledge of identification methods and predictive control. It is questionable, whether this drawback can be overcome in the future and the tuning of the predictive controller would be feasible for wide range of practitioners.

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