Analyze performances of a Discrete Controller and a Fuzzy Regulator for heating with intermittency of a dwelling

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SUMMARY

In this paper, we choose to present the generic form of an optimal control law for heating building. To build it, three successive steps are necessary: (i) the first one allows to determine the inside temperature set point that takes into account intermittency and tends to reduce the cost control; (ii) the second that corresponds to the discrete controller design is based on a quadratic mixed criterion “cost and efficency”; (iii) the third corresponds to the design of the virtual sensor. Efficiency of this approach is shown via a numerical example and compared with a Fuzzy regulator (marketed regulator).

INTRODUCTION

Taking into account the intermittency of buildings’ occupation allows saving a significant amount of energy. Many studies are related to saving heating in buildings, because these ones in tertiary sector are only occupied for 30% of time. Today, it is also necessary to work on the residential sector where intermittency exists: dwelling houses being usually empty during the day and may be slowed right down at night.

One way to improve these installations is working downstream of heating sources and defining a strategy to minimize the energy required for heating the dwelling houses. This last one has to take into explicit account of intermittency by stressing the passive contributions and storage in vacant periods. Thus during the day, solar contributions must be exploited as much as possible and energy storage must be optimized in order to use it during evening, when the outside temperature is the less favourable.

An early study in continuous time [1] has already shown the interest of an optimal control strategy (via simulation) [2,3]. In order to take into account technical considerations such as the choice of sampling, we propose to develop in discrete time a similar strategy, based on optimal control for sampled systems. Searching an extremum for the discrete performance criterion leads to a command in the form of recursive equations ready to set up. In [1], a Feed-Forward using direct measurement of outside temperature is also used. To take into account solar contribution, Feed Forward needs an outside sensor and a model of behaviour between solar radiance and inside temperature. This last task is hard to carry out because of the non stationary and non linear feature of solar radiance and also due to solar contributions trough wall and windows. Indeed these last ones show significant difference in term of dynamic behaviour. Thus, we proposed here an elegant solution of this problem.

First, a cost criterion is used to determine the optimal trajectory of inside temperature during inoccupation stage. Searching an optimum of this criterion leads naturally to define the right time to come back to the occupation set point. Thus, a complete profile of temperature set point is built taking into consideration the specific constraints of intermittency (high and low set point).
Second, the structure of the regulator is obtained using a performance criterion composed of quadratic terms related to the quality of the heating control and cost.
Third, a virtual sensor is defined to estimate states required to control the system and the equivalent control that will be delivered by an exact real dynamic Feed-Forward using a direct measurement of disturbances. However, the virtual sensor uses only measurement of inside temperature (call virtual dynamic feed-forward later). It uses an optimal estimator in sense of the minimum variance of estimation error. This step is treated as a deconvolution problem.
Last, to show efficiency of this approach, a numerical example is treated considering only the outside temperature as an outside source. After having presented the cell test (Minibat [4]) and the behaviour model we selected, our new approach will be compared with an existing fuzzy regulator (marketed regulator). To end, we will compare our virtual Feed Forward with an exact real dynamic Feed Forward.

METHOD DEVELOPPMENT

In this part, we choose to present the generic form of the control law. To build it, three successive steps are necessary: (i) the first one allows to determine the inside temperature set point that takes into account intermittency and tends to reduce the cost control; (ii) the second that corresponds to the discrete controller design is based on a quadratic mixed criterion “cost and efficency” [2,3]; (iii) the third corresponds to the design of the virtual sensor [5].

Let us consider the general representation of state of the sampled linear systems, as follows:

$$\begin{aligned} x_{k+1} &= Fx_k + Gu_k \\ y_k^m &= Cx_k + d_k \end{aligned} \tag{1}$$

With x_k the state vector, u_k the control vector, y_k^m the measurement, d_k the contribution of outside source. F, G and C are matrices with appropriate dimensions.
It is right to consider that the inside temperature of a dwelling is the sum of all sources’ contributions (heating device, outside temperature or solar radiance). Thus the corresponding model d_k will be considered in a first assumption as a white stochastic process, non-zero mean. In this paper, the variance of such process will be considered as constant.

Step 1 – Intermittency and set point profile

In this step, as we are interested in an ideal set point profile, a representation without disturbance must be considered:

$$\begin{aligned} x_{k+1} &= Fx_k + Gu_k \\ y_k &= Cx_k \end{aligned} \tag{2}$$

The aim is to define the optimal couple (u^*,y^*) for the inoccupation phase respecting specific constraint previously stated. So a quadratic cost criterion is used

$$J_1 = 0.5\left[y_{N_2}^r - y_{N_2}\right]^T Q_{N_2}\left[y_{N_2}^r - y_{N_2}\right] + 0.5\sum_{k=N_1}^{N_2-1} u_k^T R_k u_k \tag{3}$$

With N_1 and N_2 the first and the last sample of inoccupation phase. R_i and Q_i are matrices respectively definite positive and semi-definite positive. The first term of the criterion is used to find the optimal restart instant. The second allows the determination of the output trajectory that requires the lowest command.

Let us consider now the cost function Φ_k

$$\Phi_k\left(u_k\right) = 0.5 u_k{}^T R_k u_k \tag{4}$$

And the Hamiltonian H_k such as:

$$H_k = -\Phi_k + \beta_{k+1}^T\left[F x_k + G u_k\right] \tag{5}$$

with the adjoin state β_k.

Using the Discret Maximum Principe [3], we obtain the minimal control u_k^* :

$$\beta_k = -\frac{\partial H}{\partial x_k} \quad \Rightarrow \quad \beta_k = F^T \beta_{k+1} \tag{6}$$

$$\frac{dH}{du_k} = 0 \Rightarrow \; u_k^* = R_k^{-1} G^T \left(F^T\right)^{N_2-k-1} \beta_{N_2} \tag{7}$$

Using (7) in (1), some manipulations lead to:

$$\beta_{N_2} = \left(\sum_{k=N_1}^{N_2-1} F^k G R_k^{-1} G^T \left(F^T\right)^k\right)^{-1} \left(I - F^{N_2}\right) x_{N_2}^r \tag{8}$$

with $x_{N_2}^r$ the wished final value of state.

Using (7), (8), we obtain the ideal set point trajectory y_k^* for the inoccupation phase:

$$x_{k+1}^* = F x_k^* + G R_k^{-1} G^T \left(F^T\right)^{N_2-k-1} \left(\sum_{k=N_1}^{N_2-1} F^k G R_k^{-1} G^T \left(F^T\right)^k\right)^{-1} \left(I - F^{N_2}\right) x_{N_2}^r$$
$$y_k^* = C x_k^* \tag{9}$$

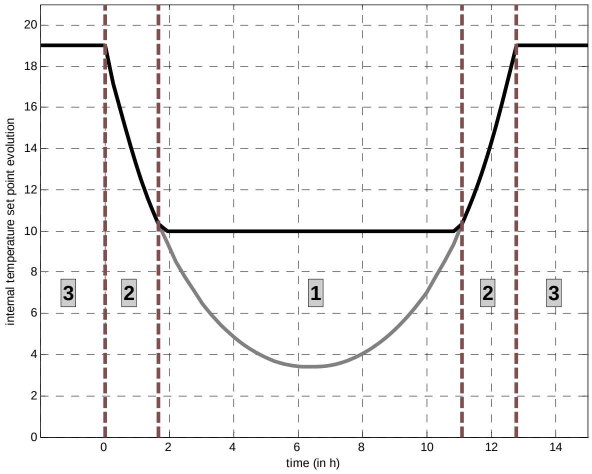

Figure 1: set point trajectory (gray – without constraint, black – with constraint)

Thus, the set point profile without constraint corresponds to a hyperbolic trajectory (9). Taking into account the specific constraint of each phase, the global trajectory of temperature is illustrated Figure 1**.** The evolution is finally composed by 3 parts. In part 2, if a regulator ensures *y**, the corresponding control is equivalent to the lowest *u**. Indeed, the dwelling is considered as a mono-zone space: the input is the heating power and the resulting output is the global inside temperature (considered as homogeneous). This SISO characteristic (Single Input Single Output) leads to a single solution. As illustrated Figure 1, constant set points are used (high and low set point) in parts 1 and 3 according to occupation and inoccupation constraints.

Step 2 – Design of control parameters

From the global set point profile defined in step 1, we are going to treat a pursuit problem in order to design the discrete controller. Thus, let us consider the following criterion:

$$J = 0.5\left[y_N^r - y_N\right]^T Q_N \left[y_N^r - y_N\right] + 0.5\sum_{k=0}^{N-1}\left\{\left[y_k^r - y_k\right]^T Q_k \left[y_k^r - y_k\right] + u_k^T R_k u_k\right\} \tag{10}$$

where y_N^r is the high set point and N is the total number of samples included in the whole cycle.

In order to ensure the reduction of the effect due to the disturbance, a new state z_k is introduced. That leads to an increased discrete representation equivalent to the representation of Johnson into the continuous case [1].

$$\begin{gathered} X_{k+1} = \begin{bmatrix} x_{k+1} \\ z_{k+1} \end{bmatrix} = F_a X_k + G_a u_k^c \quad \text{with} \quad F_a = \begin{bmatrix} F & G \\ 0 & I \end{bmatrix} \\ Y_k = C_a X_k \quad \text{with} \quad C_a = \begin{bmatrix} C & 0 \end{bmatrix} \text{ and } G_a = \begin{bmatrix} 0 \\ I\ \mathrm{T_e} \end{bmatrix} \end{gathered} \tag{11}$$

with Te the sample rate, I the identity matrix and subscript "a" to identify the increased representation for control. We introduce the Income function Ω_k defined by:

$$\begin{gathered} \Omega_k = 0.5\left[X_k^r - X_k\right]^T C_a^T Q_k^a C_a \left[X_k^r - X_k\right] + 0.5\, u_k^{cT} R_k^a u_k^c \\ \text{with} \quad Q_k^a = \begin{bmatrix} Q_k & 0 \\ 0 & R_k \end{bmatrix} \text{ and } R_k^a = \theta_k \end{gathered} \tag{12}$$

with θ_k a weighting matrix, definite positive and symmetrical.
The optimal value of the criterion is defined by a recurring form of the type:

$$J_k^* = \min_{u_k^c}\left(\Omega_k + J_{k+1}^*\right) \tag{13}$$

Where J_k^* is supposed to be a quadratic form. Thus, we define it as follows:

$$J_{k+1}^* = 0.5\left(X_{k+1}^T P_{k+1} X_{k+1} + 2\lambda_{k+1}^T X_{k+1}\right) \tag{14}$$

With λ_k a adjoin vector and P_k a definite positive and symmetrical matrix. By taking account (11), (13) and (14), the minimum of the criterion (10) with respect to u_k^c is defined as follows:

$$u_k^{c*} = -\left(R_k^a + G_a^T P_{k+1} G_a\right)^{-1} G_a^T \left(P_{k+1} F_a X_k + \lambda_{k+1}\right) = -K_k^c X_k + L_k \lambda_{k+1} \tag{15}$$

As J_k^* is a quadratic form that implies:

$$P_k = \left(F_a - G_a K_k^c\right)^T P_{k+1}\left(F_a - G_a K_k^c\right) + K_k^{cT} R_k^a K_k^c + C_a^T Q_k^a C_a \quad \text{with} \quad P_N = C_a^T Q_{N_1}^a C_a \tag{16}$$

$$\lambda_k = \left(F_a - G_a K_k^c\right)^T \lambda_{k+1} - C_a^T Q_k^a C_a X_k^r \qquad \text{with} \quad \lambda_N = C_a^T Q_{N_1}^a C_a^T X_{N_1}^r \tag{17}$$

Let us note that these two last equations must be solved backward in time. We must take care to solve them before using the controller. The real command of the process is then defined by:

$$u_k^* = z_k \qquad \rightarrow \qquad u_{k+1}^* = u_k^* + T_e\left[L_k \lambda_{k+1} - K_k^c \begin{bmatrix} x_k \\ u_k^* \end{bmatrix}\right] \tag{18}$$

Step 3 – Design of virtual sensor

Considering (1) and step 2, one can notice that the problem is equivalent to use a LQG approach [3]. At this step of design, x_k is required but not available, only the measurement y_k^m is available. Moreover, (1) is a stochastic process, thus, it is necessary to use an estimator to obtain an estimate of real trajectories followed by the states of the system [5]. We remind that our aim is the design of an estimator which estimates both the states and an equivalent contribution of disturbances. Thus, after introducing a deconvolution problem, we summarize a Kalman estimator.

We first define an equivalent system (with equivalent output) in which disturbance is considered as a stochastic unknown input.

$$\begin{array}{l} x_{k+1} = Fx_k + Gu_k \\ y_k^m = Cx_k + d_k \end{array} \rightarrow \begin{array}{l} x_{k+1} = Fx_k + Gu_k + Gw_k \\ y_k^m = Cx_k + v_k \end{array} \tag{19}$$

Where w_k is a non zero mean white noise with constant variance and v_k the noise measurement considered as a zero mean white noise with constant variance.
To overcome any estimation bias, we can consider that w_k is the output of a Wiener process generator driven by a zero mean Gaussian white noise with constant variance. That lead to the following representation:

$$\begin{array}{l} x_{k+1}^e = \begin{bmatrix} x_{k+1} \\ w_{k+1} \end{bmatrix} = F_e x_k^e + G_e u_k + M_e \omega_k \text{ with } F_e = \begin{bmatrix} F & G \\ 0 & I \end{bmatrix}, G_e = \begin{bmatrix} G \\ 0 \end{bmatrix} \text{ and } M_e = \begin{bmatrix} 0 \\ I\ \mathrm{T_e} \end{bmatrix} \\ y_k^m = C_e x_k^e \text{ with } C_e = \begin{bmatrix} C & 0 \end{bmatrix} \end{array} \tag{20}$$

with subscript "e" to identify the increased representation for estimation.
From (20), an optimal estimator in sense of the minimum variance of estimation error can be defined (Kalman Theory []). Thus, in the stationary case, we obtain:

$$\Pi = F_e \Pi F_e^T + M_e Q_\omega M_e^T - F_e \Pi C_e^T \left[C_e \Pi C_e^T + R_v \right]^{-1} C_e \Pi F_e \tag{21}$$

With Π a definite positive and symmetrical matrix, Q_ω and R_k are respective input and measurement noise variance.

$$K_e = \Pi C_e^T \left[C_e \Pi C_e^T + R_v \right]^{-1} \tag{22}$$

With K_e the estimator gain
The "a priori" estimation is defined by:

$$\hat{x}_{k/k-1}^e = F_e\ \hat{x}_{k-1/k-1}^e + G_e u_{k-1} \tag{23}$$

And the "a posteriori" estimation is defined by:

$$\hat{x}_{k/k}^e = \hat{x}_{k/k-1}^e + K_e \left(y_k^m - C_e \hat{x}_{k/k-1}^e \right) \tag{24}$$

Step1 + Step2 + Step 3 = Control law

The final expression of control is as follows:

$$u_k = u_{k-1} + T_e \left[L_{k-1} \lambda_k - K_{k-1}^c \begin{bmatrix} \hat{x}_{k-1/k-1} \\ u_{k-1} \end{bmatrix} \right] - \hat{w}_{k/k} \tag{25}$$

RESULTS

Performances of the new controller are shown thanks to the model [] of the test cell Minibat developed by the CETHIL [4]. This test installation is a 2 identical contiguous area with a controlled climatic environment (temperature, sunning). The rooms dimensions are $3.10\ m \times 3.10\ m \times 2.50\ m$ (L x l x h). The external jacket consists of standard insulating concrete (Siporex) with *20 cm* thickness and has following dimensions: $7.5\ m \times 4.50\ m \times 3.43\ m$. Walls of the cell are composed of agglomerated wood panels of *5 cm* thick, covered with a pressure-sealed plasterboard of *1 cm*. Only the southern wall is equipped with a window of *1 cm* thickness. The floor is made of a concrete flagstone of *20 cm* thickness. A set of runs was carried out with the cell in order to identify its dynamic and static behaviour. A first order continuous model is obtained:

$$F = e^{-\frac{T_e}{\tau}}, \quad G = b\left[1 - e^{-\frac{T_e}{\tau}}\right] \quad C = 1 \qquad (26)$$

$$\text{with } \; T_e = 1000 \text{ s}, \; \tau = 9564\text{s} \; \text{ and } \; b = 0.008413$$

F, G and C are defined as in (1).

In our case, the outside temperature is the only disturbance considered. The weather data come from the Météonorm station located at Lyon – Bron, France. Simulation runs on two days with occupation range 8h00-19h00

Figure 2 shows results we obtained with a controller using an optimal control strategy combined with a real dynamic Feed-Forward (RFF) when all information are known (states and disturbance). Figure 2 shows also results we obtained with the same kind of control strategy but information needed is estimated with a virtual sensor. It clearly appears that performances of both controllers are quite similar.

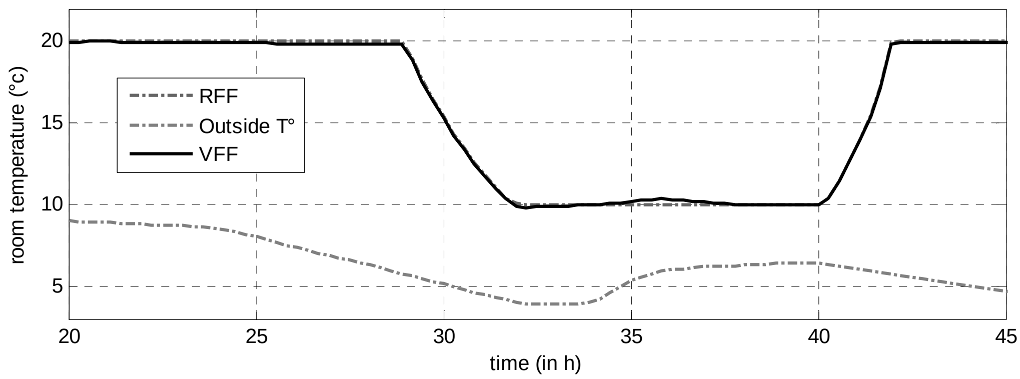

Figure 2 ***: Comparison of our controller (VFF) with an optimal control with all states available and real dynamic feed forward (RFF)***

Figure 3 gives evolution of inside temperature we obtained with a fuzzy controller [6, 7, 8, 9], and our discrete controller. Our new strategy makes it possible to satisfy with a great efficiency the constraints. Let us consider now the first inoccupation phase on Figure 3 and Figure 4. When the inoccupation phase starts, the Fuzzy controller is designed to deliver a constant control whatever external contributions. As a result, this technique spends more energy than necessary. In our case, the control is automatically reduced or stopped when no energy is required and restarts automatically when the low set point is not satisfied. Moreover, for the Fuzzy controller, the time to come back is defined a priori by the operator and the associated control is the maximum control. Thus, room temperature is equivalent to the high set point too much early. As a conclusion, energy expenditure with the Fuzzy controller is more important.

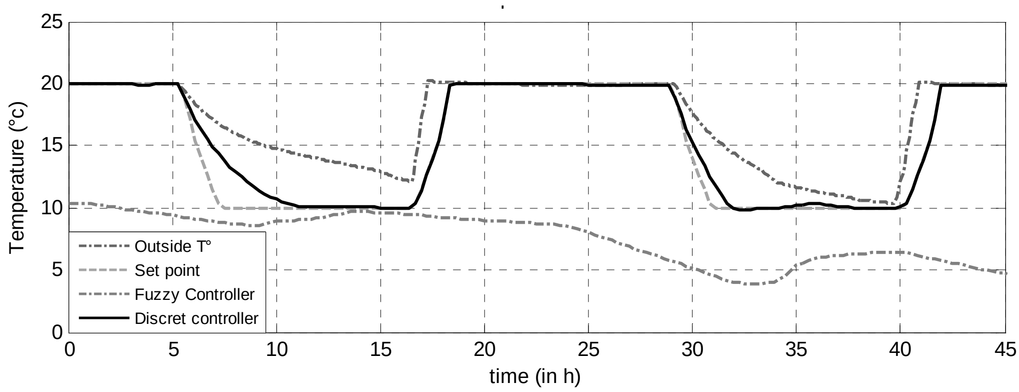

Figure 3 ***: Comparison of our controller (VFF) with a Fuzzy controller during two days (room temperature)***

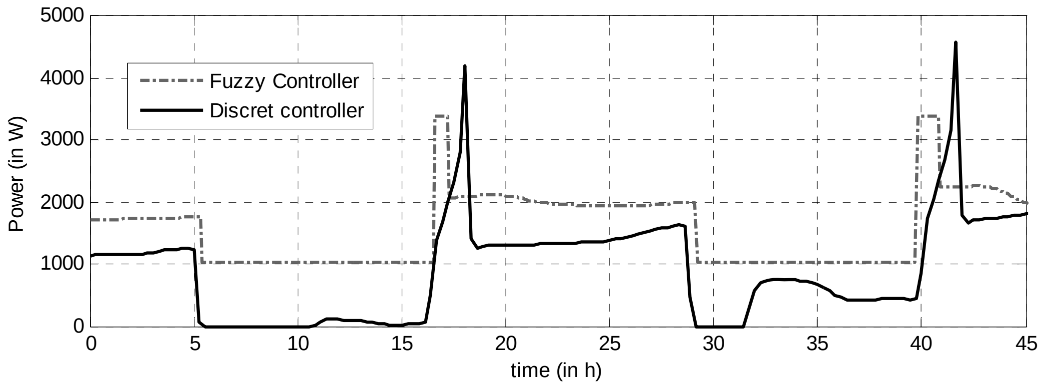

Figure 4 ***: Comparison of our controller (VFF) with a Fuzzy controller during two days (control)***

CONCLUSION

In this article, the synthesis of a discrete controller for the control of the temperature in a building was presented. After introduction, a Cost criterion makes it possible to characterize the specific behaviour to each phase (occupation and inoccupation). Parameters of the control law are next obtained using an optimal control analysis from a Cost-Efficiency criterion. A virtual sensor is then defined to take into account outside contributions from only room temperature measurement. Results of simulations are particularly hopeful and show, in comparison with a fuzzy regulator, the interest to use a model of behaviour of the test cell and more generally of a building. Our controller allows to decrease significantly the amount of energy needed to heat the building.

RÉFÉRENCES

1 **Raffenel,** Y., Choulak, S., Blanco, E., Virgone, J., Commande optimale basée sur le modèle d'un bâtiment pour le chauffage intermittent, *SFT 2006*, 16-19 Mai 2006,6p

2 **Ramirez**, W.F., Process Control and Identification, *Academic Press*, 1994

3 **Anderson**, B. D. O., **Moore**, J.B., Optimal control – Linear Quadratic Methods, *Prentice Hall*, 1990

4 **Kuznik**, F., Virgone, J., Noël, J., Etude dynamique d'une cavité contrôlée soumise à diverses sollicitations thermiques – expérimentation et modélisation, *journées internationales de thermique, JITH*, Tanger 15-17 novembre 2005, p 363-366.

5 **Gelb**, A., Applied optimal estimation, *MIT Press*. 1974

6 **Fraisse** G., La régulation thermique des bâtiments tertiaires : application de la logique floue à la régulation centrale du chauffage en régime intermittent, *thèse de docteur-ingénieur*, INSA de Lyon, 1997
7 **Virgone** J., Fraisse G., Optimisation of intermittent heating: experimental results of a new remote controller,*Clima 2000, Napoli 2001 world congress*, 15-18 septembre 2001, 13p.
8 **Fraisse,** G., Virgone J., Roux, J.J., Thermal Control of a discontinuously occupied building using a classical and a fuzzy logic approach, *Energy and Buildings* 26 (1997) 303-316
9 **Kolokotsa**, D. & Al, Fuzzy control for improved buildings energy management systems, *European Symposium on Intelligent Techniques*, June 3-4, 1999, Greece