

# A dynamic horizon distributed predictive control approach for temperature regulation in multi-zone buildings

Petru-Daniel Moroşan, Romain Bourdais, Didier Dumur and Jean Buisson

**Abstract**—This paper proposes a distributed predictive control strategy for building temperature regulation. The originality of the approach consists of using a dynamic prediction horizon MPC, allowing a more effective disturbances rejection. Then this idea is extended, for multizone temperature control in buildings, in a distributed manner, where each controller solves a local optimization problem using information about the expected behavior of the other local regulators. The effectiveness of the proposed approach is showed through different simulations.

## I. INTRODUCTION

Nowadays the decrease of the energy consumption is a world objective and it is no longer feasible to design a system without concerning the energy optimization. The building heating system is an important energy consumer, being responsible of more than 20% of the total energy consumption in Europe. Even if the new trends are to construct buildings to meet new environmental standards, the problem remains unsolved for older buildings where thermal insulation works are difficult. For these later, only an optimized use of existing heating systems can reduce the energy bill. This is the context in which the present work takes place.

In all kind of building, time scheduled controllers are more and more used, because they can adjust the indoor temperature according to occupation profile which plays a crucial role in this energy optimization. In order to obtain a desired temperature at a given time, it seems obvious to turn on the heating system with anticipation which addresses the following question: what is the optimal moment to trigger the system and what is the required power to reach the temperature reference? In practice, it is often the experience which enables this setting, but the optimal moment to turn on the heating system can vary between some minutes and several hours before the beginning of an occupation period, depending on the outdoor temperature and other varying disturbances. The aim of this work is to propose a control structure that overcomes this problem.

To allow the anticipative effect, the use of a predictive control law seems relevant. Such a choice is motivated by the fact that in practice, the future occupation profile can be known or well estimated. In a few words, this technique consists in computing the control input sequence as the result

of an optimization of a given criterion which includes some future information (occupation profile and desired temperature reference). In the heating system case, the minimization criterion optimizes a compromise between the occupants comfort and energy consumption. This technique has been used in many works [1], [2], [3].

In this paper, we propose an original approach based on the basic principles of the predictive control. But we include here an additional parameter, the occupation profile of the rooms, which leads to define a dynamic criterion, as in [4]. The larger the prediction horizon is, the better the anticipation is optimized, but as it is well-known in the predictive control community, a large prediction horizon implies less efficient disturbance rejections. The developed idea is then to solve an optimization problem with a variable horizon size. To adapt this control structure to large scale systems (buildings with many rooms for instance), different solutions can be proposed. Using a decentralized strategy, each room is controlled independently. This approach does not take into account the thermal influences between rooms, so the global performance will be decreased. A centralized control scheme, in which these influences are considered in the control model, reaches the optimality, but the number of variables in this optimization introduces a combinatorial explosion of the computation load. In this paper we present a distributed predictive control structure (dMPC). The air temperature of the rooms is regulated by a local controllers, which communicate with the others. This structure combines the advantages of the previous ones: a result close to the optimal solution with less computational demand.

The paper is constructed as follows. In Section II, the dynamic prediction horizon is formalized. The efficiency of the method is then illustrated by simulation. In Section III, the approach developed for one zone is extended to large scale buildings, highlighting the advantages of a distributed control strategy. The convergence of the control algorithm is analyzed. The performances of the method are illustrated by simulations. Conclusions and perspectives are presented in Section IV.

## II. DYNAMIC PREDICTION HORIZON. SINGLE ROOM CASE

Heating systems are characterized by strong inertia and very slow dynamics. Thus, in order to ensure a certain comfort temperature at the beginning of an occupation period it is necessary to have a large prediction horizon. This also provides better stability margins of the closed loop system, but the control input becomes less aggressive and then less robust to disturbances [5]. In the heating case,

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the indoor temperature fluctuations are frequent and have relatively faster dynamics (solar radiation, door or window openings, ...). To have a better rejection of these disturbances (assuming that their behavior is unknown), it becomes necessary to have a reduced prediction window. To avoid sudden variations of prediction horizon from one sample time to another we use the following idea: as soon as we enter into a transition Inoccupation-Occupation (the occupation vector becomes  $\delta^k = [0 \ \dots \ 0 \ 1]^T$ ) the window size is gradually reduced (up to a minimum value  $N_2^{min}$ ). We return to the maximum prediction horizon  $N_2^{max}$  as soon as we re-enter in the inoccupation period. The dynamic behavior of the prediction window size is illustrated by the Fig. 1 and can be formalized by the following equation:

$$N_2^c(k) = \begin{cases} N_2^{max} & , \delta^k(j) = 0, \forall j = 1 \dots N_2 \\ j & , j \text{ is the maximum which satisfies} \\ & \delta^k(j) > \delta^k(j-1) \text{ and } j \geq N_2^{min} \\ N_2^{min} & , j \text{ is the maximum which satisfies} \\ & \delta^k(j) > \delta^k(j-1) \text{ and } j < N_2^{min} \\ & \text{or } j \in \emptyset \text{ and } \delta^k(1) = 1. \end{cases}$$

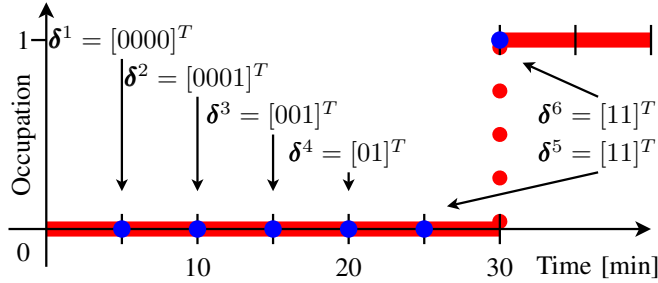


Fig. 1. Evolution of the prediction horizon size ( $N_2^{min} = 2$ ,  $N_2^{max} = 4$ ,  $T_s = 10\text{min}$ )

### A. Control law design

The first step in a predictive control approach is to define the prediction model. The heating process can be described by a linear model, represented in a discrete state space (with the sampling time  $T_s$ ) as:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}u(k) \\ y(k) = \mathbf{C}\mathbf{x}(k) \end{cases} \quad (1)$$

where the vector  $\mathbf{x} \in \mathbb{R}^{n \times 1}$  is the state,  $u, y \in \mathbb{R}$  are the input (heating power) and the output (measured room temperature), respectively.

From the model (1), we can write the output predicted at time step  $k+i$ , from the current state  $\mathbf{x}(k)$  as:

$$\hat{y}(k+i|k) = \mathbf{C}\mathbf{A}^i\mathbf{x}(k) + \sum_{j=0}^{i-1} \mathbf{C}\mathbf{A}^{i-j-1}\mathbf{B}u(k+j). \quad (2)$$

Considering (2) for  $i = 1 \dots N_2^c(k)$ , the output predicted sequence becomes:

$$\hat{\mathbf{y}}(k) = \mathbf{\Psi}^k\mathbf{x}(k) + \mathbf{\Phi}^k\mathbf{u}(k),$$

with the matrices

$$\mathbf{\Psi}^k = [(\mathbf{C}\mathbf{A}^1)^T \ \dots \ (\mathbf{C}\mathbf{A}^{N_2^c(k)})^T]^T, \\ \mathbf{\Phi}^k = \begin{bmatrix} \phi^0 & 0 & \dots & 0 \\ \phi^1 & \phi^0 & 0 & \dots \\ \dots & \dots & \ddots & \dots \\ \phi^{N_2^c(k)-1} & \dots & \phi^{N_2^c(k)-N_u+1} & \sum_{k=0}^{N_2^c(k)-N_u} \phi^k \end{bmatrix}, \\ \phi^k = \mathbf{C}\mathbf{A}^k\mathbf{B},$$

and the vectors

$$\hat{\mathbf{y}}(k) = [\hat{y}(k+1|k) \ \dots \ \hat{y}(k+N_2^c(k)|k)]^T, \\ \mathbf{u}(k) = [u(k) \ \dots \ u(k+N_u-1)]^T.$$

We define now a dynamic criterion which takes in account the future occupation profile: during the occupation period, both comfort and energy consumption have to be optimized, whereas during the inoccupation period only the energy consumption is minimized. Based on these remarks, the cost function can be expressed as:

$$\begin{aligned} J^k &= (\mathbf{w}(k) - \hat{\mathbf{y}}(k))^T \mathbf{\Delta}^k (\mathbf{w}(k) - \hat{\mathbf{y}}(k)) + \mathbf{u}(k)^T \mathbf{\Lambda}^k \mathbf{u}(k) \\ &= \mathbf{u}(k)^T (\mathbf{\Phi}^{kT} \mathbf{\Delta}^k \mathbf{\Phi}^k + \mathbf{\Lambda}^k) \mathbf{u}(k) \\ &\quad - 2(\mathbf{w}(k) - \mathbf{\Psi}^k \mathbf{x}(k))^T \mathbf{\Delta}^k \mathbf{\Phi}^k \mathbf{u}(k) \\ &\quad + (\mathbf{w}(k) - \mathbf{\Psi}^k \mathbf{x}(k))^T \mathbf{\Delta}^k (\mathbf{w}(k) - \mathbf{\Psi}^k \mathbf{x}(k)), \end{aligned}$$

where

$$\mathbf{w}(k) = [w(k+1) \ \dots \ w(k+N_2^c(k))]^T, \\ \mathbf{\Delta}^k = \text{diag}\{\delta^k(1), \delta^k(2), \dots, \delta^k(N_2^c(k))\}, \\ \mathbf{\Lambda}^k = \text{diag}\{\lambda, \dots, \lambda, (N_2^c(k) - N_u + 1)\lambda\}.$$

The dynamical behavior of this criterion is represented by the occupation vector  $\delta^k$  defined by  $\delta^k = [\delta^k(1) \ \dots \ \delta^k(N_2)]^T$ , where  $\delta^k(j) = 1$  if  $k+j \in \text{Occupation}$  and  $\delta^k(j) = 0$  otherwise. The optimal control input sequence at time step  $k$  can be analytically expressed in the unconstrained case as:

$$\mathbf{u}^{opt}(k) = (\mathbf{\Phi}^{kT} \mathbf{\Delta}^k \mathbf{\Phi}^k + \mathbf{\Lambda}^k)^{-1} \mathbf{\Phi}^{kT} \mathbf{\Delta}^k (\mathbf{w}(k) - \mathbf{\Psi}^k \mathbf{x}(k)).$$

For practical reasons and to reduce the complexity of the algorithm, we choose  $N_2^{min} = N_u$ . In this particular case,  $\mathbf{\Psi}^k$  and  $\mathbf{\Phi}^k$  do not need to be computed on line. During the initialization phase of the control algorithm, these two matrices are calculated for  $N_2^{max}$  and the required values at each sample time are obtained by selecting the first  $N_2^c(k)$  lines of these matrices.

### B. Simulation results

The results presented in this section have been obtained using the MATLAB toolbox called SIMBAD [6]. The simulated room has a  $42\text{m}^3$  volume. It is equipped with an electric convector of  $1200\text{W}$  maximum power. Two performance indices have been defined to compare the proposed strategy with a fixed horizon MPC. They represent the energy consumption and the comfort:

- The energy, in kWh, consumed for indoor heating:

$$I_W = \sum_{k=k_0}^{k_f} u(k)T_s.$$

- The comfort index penalizes the difference between the measured room air temperature and the reference, but only during the occupation periods. In °Ch:

$$I_C = \sum_{k=k_0}^{k_f} |w(k) - y(k)| T_s \delta^{k-1} (1).$$

From a qualitative point of view, Fig. 2 highlights the better disturbance rejection of the MPC controller with dynamic prediction horizon. Quantitatively, table I shows the results obtained simulating a period of one month. The external meteorological conditions simulated were those measured in Rennes, in January 1998. If the predictive controllers are more efficient than the on/off regulator, the dynamic horizon one is even better.

TABLE I  
PERFORMANCE COMPARISON

Control law	$I_C$ [°Ch]	$I_W$ [kWh]
On/Off	369	206
MPC with $N_2$ fixed	134	196
MPC with $N_2$ dynamic	100	191

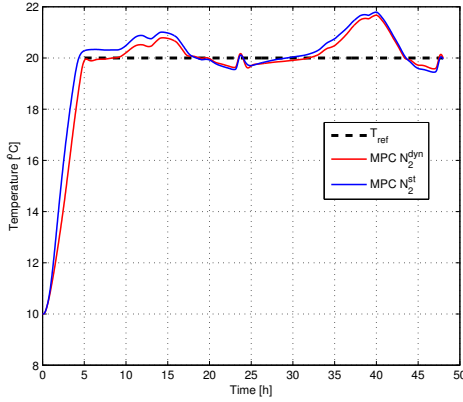


Fig. 2. MPC with fixed and dynamic prediction horizon

### III. MULTI-ZONE EXTENSION

A question arises naturally: how to extend this approach to a multi-zone building? The most natural way would be to apply this method in a room by room approach, so-called *decentralized* method. Each controller acts independently. The problem with this approach is that it does not take into account the thermal coupling between rooms, through walls and air circulation due to an open door, which leads in a loss in control performance.

Another solution would be to take into account these coupling effects in the prediction model and to define a global regulator that controls all the rooms. This approach, so-called

*centralized* approach, improves the system performances, but the computational complexity due to the number of optimization variables, grows exponentially with the number of zones. On the other hand, if the central controller fails, the entire building is penalized.

Therefore, we propose a *distributed* approach where the building heating system is controlled by local regulators which communicate. Using the communication network, the controllers exchange information regarding their future behavior. This information sharing allows the distributed scheme to converge towards the global optimal solution [7], [8] or towards a Nash equilibrium [9], [10] with a reduced computational load. A performance comparison of the three control strategies for temperature regulation in multi-zone buildings (decentralized, centralized and distributed) in a MPC framework can be found in [11].

The main contribution of this paper is presented in the next section, in which the dynamical horizon distributed approach is formalized. In this case, the quantity of information exchanged depends on the size of the prediction horizon.

#### A. Formalization of the dynamical horizon distributed MPC

As we have to consider the thermal coupling between the rooms, the previous model (1) has to be adapted. For one room  $i$  the thermal influences of the neighbor rooms are represented by an output coupling. This leads to the following equation:

$$\begin{cases} \mathbf{x}_i(k+1) = \mathbf{A}_i \mathbf{x}_i(k) + \mathbf{B}_{ii} u_i(k) + \sum_{j \in \tilde{h}_i} \mathbf{B}_{ij} y_j(k) \\ y_i(k) = \mathbf{C}_i \mathbf{x}_i(k) \end{cases} \quad (3)$$

where  $\tilde{h}_i$  is the set of adjacent rooms of the zone  $i$  and  $\mathbf{x}_i \in \mathbb{R}^{n_i}$ ,  $u_i, y_i \in \mathbb{R}$  are respectively the local state, the control input and the output.

Note that this model structure can be derived by considering only the convective heat transfer between two adjacent zones [12] and ignoring the external perturbations. From (3), the output prediction equation for the subsystem  $i$  can be expressed as:

$$\hat{y}_i(k) = \Psi_i^k \mathbf{x}_i(k) + \Phi_{ii}^k u_i(k) + \sum_{j \in \tilde{h}_i} \Phi_{ij}^k \tilde{y}_j(k), \quad (4)$$

with the following notations

$$\begin{aligned} \hat{y}_i(k) &= [\hat{y}_i(k+1|k) \quad \dots \quad \hat{y}_i(k+N_{2i}^c(k)|k)]^T, \\ \mathbf{u}_i(k) &= [u_i(k|k) \quad \dots \quad u_i(k+N_{ui}-1|k)]^T, \\ \tilde{y}_j(k) &= [\tilde{y}_j(k) \quad \tilde{y}_j(k+1) \quad \dots \quad \tilde{y}_j(k+N_{2j}^{cmax}(k)-1)]^T \\ N_{2i}^{cmax}(k) &= \max_{j \in \tilde{h}_i} N_{2j}^c(k), \end{aligned}$$

$$\begin{aligned} \Psi_i^k &= [\psi_i^{1T} \quad \dots \quad \psi_i^{N_{2i}^c(k)T}]^T, \\ \Phi_{ii}^k &= \begin{bmatrix} \phi_{ii}^0 & 0 & \dots & 0 \\ \phi_{ii}^1 & \phi_{ii}^1 & 0 & \dots \\ \dots & \dots & \ddots & \dots \\ \phi_{ii}^{N_{2i}^c(k)-1} & \dots & \phi_{ii}^{N_{2i}^c(k)-N_{ui}+1} & \sum_{k=0}^{N_{2i}^c(k)-N_{ui}} \phi_{ii}^k \end{bmatrix}, \end{aligned}$$

$$\Phi_{ij}^k = \begin{bmatrix} \phi_{ij}^0 & 0 & \cdots & \cdots & 0_{1 \times N_{2j}^{cmax}(k) - N_{2i}^c(k)} \\ \phi_{ij}^1 & \phi_{ij}^0 & 0 & \cdots & 0_{1 \times N_{2j}^{cmax}(k) - N_{2i}^c(k)} \\ \cdots & \cdots & \ddots & \cdots & \cdots \\ \phi_{ij}^{N_{2i}^c(k)-1} & \cdots & \phi_{ij}^1 & \phi_{ij}^0 & 0_{1 \times N_{2j}^{cmax}(k) - N_{2i}^c(k)} \end{bmatrix},$$

$$\psi_i^k = \mathbf{C}_i \mathbf{A}_i^k, \quad \phi_{ij}^k = \psi_i^k \mathbf{B}_{ij}.$$

The sequence of the optimal control inputs  $\mathbf{u}_i^{opt}(k)$  is obtained by the analytic solution of the following local unconstrained minimization problem:

$$J_i^k = \|\hat{\mathbf{y}}_i(k) - \mathbf{w}_i(k)\|_{\Delta_i^k}^2 + \|\mathbf{u}_i(k)\|_{\Lambda_i^k}^2 \quad (5)$$

and is

$$\mathbf{u}_i^{opt}(k) = \Xi_i^k \left( \mathbf{w}_i(k) - \Psi_i^k \mathbf{x}_i(k) - \sum_{j \in \bar{h}_i} \Phi_{ij}^k \tilde{\mathbf{y}}_j(k) \right),$$

with  $\Xi_i^k = (\Phi_{ii}^{kT} \Delta_i^k \Phi_{ii}^k + \Lambda_i^k)^{-1} \Phi_{ii}^{kT} \Delta_i^k$ .

The key point of our distributed approach consists in the way of constructing the output prediction at a time step  $k$ . To do this, we use the predicted future behavior of the neighbors computed at the previous sample time  $k-1$  as initialization. This is the information exchanged by the controllers. As a dynamic prediction horizon is used, the prediction window size at time  $k$  can be greater than the previous prediction window ( $N_{2j}^c(k-1) < N_{2i}^c(k)$ ). In this case the vector  $\tilde{\mathbf{y}}_j(k)$  has not the required dimension. This problem can be solved using the fact the occupancy profiles are known for the maximum prediction horizon, so the window sizes can also be known in advance. This calculation can be done as follows:

$$\hat{N}_{2i}^c(k+1|k) = \begin{cases} N_{2i}^{max} & , \delta_i^k(N_{2i}^{max}) = 0 \\ N_{2i}^c(k) - 1 & , N_{2i}^c(k) > N_{2i}^{min} \text{ and} \\ & \delta_i^k(N_{2i}^{max}) = 1 \\ N_{2i}^{min} & , N_{2i}^c(k) = N_{2i}^{min} \text{ and} \\ & \delta_i^k(N_{2i}^{max}) = 1. \end{cases} \quad (6)$$

Determining  $\hat{N}_{2i}^c(k+1|k)$  for all subsystems is required in order to know the size of the vectors  $\tilde{\mathbf{y}}_j(k+1)$  which are needed by the neighbors controllers, at the next time step, to build their predictions. Each controller sends its parameter  $\hat{N}_{2i}^c(k+1|k)$  to all the neighbor regulators. Then, the sequence of future outputs is completed by a decentralized scheme, to reach the necessary dimension at the next instant. This is given by:

$$N_{2i}^{cmax}(k+1) = \max_{j \in \bar{h}_i} \hat{N}_{2j}^c(k+1|k). \quad (7)$$

The corrected vector will have the following structure:

$$\tilde{\mathbf{y}}_i(k) = [y_i(k) \quad \hat{\mathbf{y}}_i^T(k) \quad \mathbf{w}_i^T(k) \quad \bar{\mathbf{y}}_i^T(k)]^T, \quad (8)$$

containing four parts, where:

- 1)  $y_i(k)$  is the current measured output value,
- 2)  $\hat{\mathbf{y}}_i(k)$  corresponds to the local future output sequence obtained by minimizing the local cost function  $J_i^k$ ,

- 3)  $\mathbf{w}_i(k) = [w_i(k + N_{2j}^c(k) + 1) \quad \cdots \quad w_i(k + N_{oi}(k))]^T$  is the future reference sequence, with  $k + N_{oi}(k)$  the last occupation time instant,
- 4)  $\bar{\mathbf{y}}_i(k) = [\bar{y}_i(k + N_{oi}(k) + 1) \quad \cdots \quad \bar{y}_i(k + N_{2i}^{cmax}(k))]^T$  which is the local free response.

The existence of the third and fourth parts is due to the fact that the neighbors of  $i$  may have a longer prediction horizon. Our choice was to suppose that the local controller  $i$  acts perfectly and the output of the subsystem perfectly matches the reference during the occupation period. If  $N_{2i}^{cmax}(k) - N_{oi}(k) > 0$ , another part is required, corresponding to an inoccupation period. In this latter case, we complete the output sequence with the free response of the subsystem from the predicted state  $\mathbf{x}_i(k + N_{2i}^c(k))$ , which gives:

$$\bar{\mathbf{y}}_i = \bar{\Psi}_i^k \hat{\mathbf{x}}_i(k + N_{2i}^c(k)|k),$$

with

$$\bar{\Psi}_i^k = [(\psi_i^1)^T \quad \cdots \quad (\psi_i^{N_{2i}^{cmax}(k+1) - N_{2i}^c(k)})^T]^T,$$

$$\hat{\mathbf{x}}_i(k + N_{2i}^c(k)|k) = \bar{\mathbf{A}}_i^k \mathbf{x}_i(k) + \bar{\Phi}_{ii}^k \mathbf{u}_i(k) + \sum_{j \in \bar{h}_i} \bar{\Phi}_{ij}^k \tilde{\mathbf{y}}_j(k),$$

$$\bar{\Phi}_{ii}^k = [\bar{\phi}_{ii}^{N_{2i}^c(k)-1} \quad \cdots \quad \bar{\phi}_{ii}^{N_{2i}^c(k) - N_{ui} + 1} \quad \sum_{j=0}^{N_{2i}^c(k) - N_{ui}} \bar{\phi}_{ii}^j],$$

$$\bar{\Phi}_{ij}^k = [\bar{\phi}_{ij}^{N_{2i}^c(k)-1} \quad \cdots \quad \bar{\phi}_{ij}^1 \quad \bar{\phi}_{ij}^0 \quad 0_{n_i \times (N_{2j}^{cmax}(k) - N_{2i}^c(k)) p_i}],$$

$$\bar{\mathbf{A}}_i^k = \mathbf{A}_i^{N_{2i}^c(k)}, \quad \bar{\phi}_{ij}^k = \mathbf{A}_i^k \mathbf{B}_{ij}.$$

Fig. 3 illustrates the main principle of the construction of the output prediction vector at a time step  $k$ , with a distributed and a decentralized part.

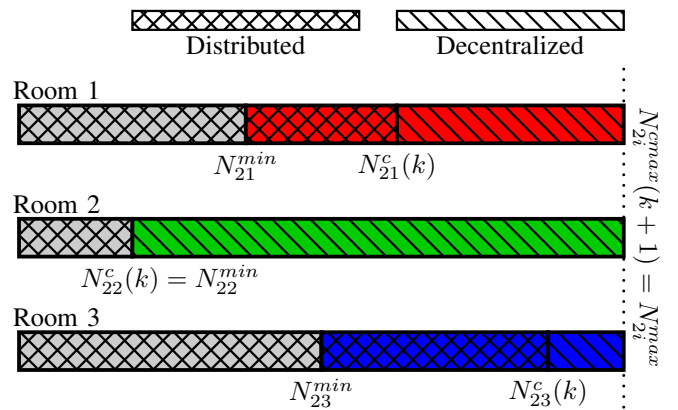


Fig. 3. Construction of the output prediction vector

The algorithm 1 synthesizes the approach presented in this section. This procedure is executed by each local controller  $i$  at each sampling time  $k$ , and can communicate several times (negotiation phase) to converge to a consensus. The stop condition is linked to the variation of the control input sequence from a negotiation step to the next one. We propose in the next section a convergence condition of the negotiation phases.

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**Algorithm 1** Dynamic prediction horizon dMPC
 

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- 1: Construct  $\tilde{\mathbf{y}}_i(k)$  using the output predictions at previous time step and the current output local measure  $y_i(k)$
  - 2: Send  $\tilde{\mathbf{y}}_i(k)$  and  $N_{2i}^c(k)$  to all neighbors  $j \in \tilde{h}_i$
  - 3: Receive  $\tilde{\mathbf{y}}_j(k)$  and  $N_{2j}^c(k)$  from all neighbors  $j \in \tilde{h}_i$
  - 4: Calculate  $N_{2i}^{cmax}(k)$ ,  $l = 0$
  - 5: **while**  $l < l_{max}$  and  $\|\mathbf{u}_i^{(l+1)}(k) - \mathbf{u}_i^l(k)\|_\infty > \epsilon$  **do**
  - 6:   Solve the local optimization  $\mathbf{u}_i^l(k) = \text{argmin} J_i^k$
  - 7:   Compute  $\hat{\mathbf{y}}_i^l(k)$ ,  $\bar{\mathbf{y}}_i^l(k)$  and construct  $\tilde{\mathbf{y}}_i^l(k)$  (9)
  - 8:   Send  $\tilde{\mathbf{y}}_i^l(k)$  to all neighbors  $j \in \tilde{h}_i$
  - 9:   Receive  $\tilde{\mathbf{y}}_j^l(k)$  from all neighbors  $j \in \tilde{h}_i$
  - 10:   Update  $l = l + 1$
  - 11: **end while**
  - 12: Apply the first element of  $\mathbf{u}_i^{(l-1)}(k)$  to local subsystem
  - 13: Calculate  $\hat{N}_{2i}^c(k+1|k)$  (6)
  - 14: Send  $\hat{N}_{2i}^c(k+1|k)$  to neighbors  $j \in \tilde{h}_i$
  - 15: Receive  $\hat{N}_{2j}^c(k+1|k)$  from all  $j \in \tilde{h}_i$
  - 16: Compute  $N_{2i}^{cmax}(k+1)$  (7),  $\bar{\mathbf{y}}_i(k+1)$  and  $\hat{\mathbf{y}}_i(k+1) = \hat{\mathbf{y}}_i^{(l-1)}(k)$ , update  $k = k + 1$  and goto step 1
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### B. Convergence analysis of the proposed distributed strategy

This section will analyze the convergence of the negotiation phase of Algorithm 1 for a given time step  $k$ . For the sake of simplicity, in the following, we will drop the  $k$ -dependencies. We can write the output sequence  $\tilde{\mathbf{y}}_j^{(l+1)}$  sent by the local controller  $j$  at a negotiation step  $l+1$  to all of its neighbors, in the most general case as:

$$\begin{aligned} \tilde{\mathbf{y}}_j^{(l+1)} &= \begin{bmatrix} y_j^T & \hat{\mathbf{y}}_j^{(l+1)T} & \mathbf{w}_j^T & \bar{\mathbf{y}}_j^{(l+1)T} \end{bmatrix}^T \\ &= \Gamma_{1j} y_j + \Gamma_{2j} \hat{\mathbf{y}}_j^{(l+1)} + \Gamma_{3j} \mathbf{w}_j + \Gamma_{4j} \bar{\mathbf{y}}_j^{(l+1)}, \end{aligned} \quad (9)$$

where all the notations are the same as the ones explained before, but specified for a given negotiation step  $l+1$ . Note that the dimensions of these vectors may change at each time step  $k$ , as shown in previous section. The prediction equation (4) at iteration  $l+1$  is:

$$\hat{\mathbf{y}}_i^{(l+1)} = \Psi_i \mathbf{x}_i + \Phi_{ii} \mathbf{u}_i^{(l+1)} + \sum_{j \in \tilde{h}_i} \Phi_{ij} \tilde{\mathbf{y}}_j^l, \quad (10)$$

and the free local response part:

$$\bar{\mathbf{y}}_j^{(l+1)} = \bar{\Psi}_j \left( \bar{\mathbf{A}}_j \mathbf{x}_j + \bar{\Phi}_{jj} \mathbf{u}_j^{(l+1)} + \sum_{s \in \tilde{h}_j} \bar{\Phi}_{js} \tilde{\mathbf{y}}_s^l \right). \quad (11)$$

The main objective of the next lines is to find a recurrence equation, expressing the dependence of the output predictions at a negotiation step  $l+1$  on the output predictions at a negotiation step  $l$ . In order to obtain a convergence condition, we use the analytical solution of the unconstrained minimization problem (5). The local optimal solution at iteration  $l+1$  is:

$$\mathbf{u}_i^{(l+1)} = -\Xi_i \sum_{j \in \tilde{h}_i} \Phi_{ij} \tilde{\mathbf{y}}_j^l + \xi_i, \quad (12)$$

with  $\xi_i = \Xi_i (\mathbf{w}_i - \Psi_i \mathbf{x}_i)$ . Replacing (10), (11) and (12) in (9) we obtain  $\tilde{\mathbf{y}}_i^{(l+1)} = \sum_{j \in \tilde{h}_i} \Theta_{ij} \tilde{\mathbf{y}}_j^l + \alpha_i$ , with the following

notations:

$$\begin{aligned} \Theta_{ij} &= \Gamma_{2i} \Phi_{ij} + \Gamma_{4i} \bar{\Psi}_i \bar{\Phi}_{ij} - (\Gamma_{2i} \Phi_{ii} + \Gamma_{4i} \bar{\Psi}_i \bar{\Phi}_{ii}) \Xi_i \Phi_{ij}, \\ \alpha_i &= \beta_i + (\Gamma_{2i} \Phi_{ii} + \Gamma_{4i} \bar{\Psi}_i \bar{\Phi}_{ii}) \xi_i, \\ \beta_i &= \Gamma_{1i} \mathbf{y}_i + \Gamma_{3i} \mathbf{w}_i + (\Gamma_{2i} \Psi_i + \Gamma_{2i} \bar{\Psi}_i \bar{\mathbf{A}}_i) \mathbf{x}_i. \end{aligned}$$

The global vector of the exchanged output sequences defined as  $\tilde{\mathbf{Y}}^{(l+1)} = \begin{bmatrix} \tilde{\mathbf{y}}_1^{(l+1)T} & \tilde{\mathbf{y}}_2^{(l+1)T} & \dots \end{bmatrix}^T$  has the following expression:

$$\tilde{\mathbf{Y}}^{(l+1)} = \Theta \tilde{\mathbf{Y}}^l + \alpha, \quad (13)$$

where  $\Theta = [\Theta_{ij}]$ , with  $\Theta_{ij} = \mathbf{0}_{N_{2i}^{cmax} \times N_{2j}^{cmax}}$ ,  $\forall j \notin \tilde{h}_i$  and  $\alpha = \text{block-diag}\{\alpha_1, \alpha_2, \dots\}$ . The equation (13) is the desired recurrence one and due to the fact that from (12) in a similar way as in (13) the global input control sequence can be expressed as  $\tilde{\mathbf{U}}^{(l+1)} = \Xi \tilde{\mathbf{Y}}^l + \xi$ , where  $\Xi = [\Xi_{ij}]$ , with  $\Xi_{ij} = \mathbf{0}_{N_{ui} \times N_{2j}^{cmax}}$ ,  $\forall j \notin \tilde{h}_i$  and  $\xi = \text{block-diag}\{\xi_1, \xi_2, \dots\}$ , then a necessary and sufficient condition can be formulated:

*Proposition 1:* (Convergence of Algorithm 1). For a given time step  $k$ , the negotiation process (described by the while loop in Algorithm 1) converges to a consensus, if and only if  $|\rho\{\Theta\}| < 1$ . That is the spectrum radius must be less than 1 to guarantee a convergent computation.

The next section presents the simulation results, illustrating not only the efficiency of the proposed approach but also the convergence mechanism.

### C. Simulation results

This algorithm has been tested in simulation using the SIMBAD toolbox, for a building with three rooms of 42m<sup>3</sup> volume, equipped with 1200W maximum power electric convectors, according to the scheme in Fig. 4. The prediction model (which attains 10 state variables) has been obtained by experimental identification of the simulation model.

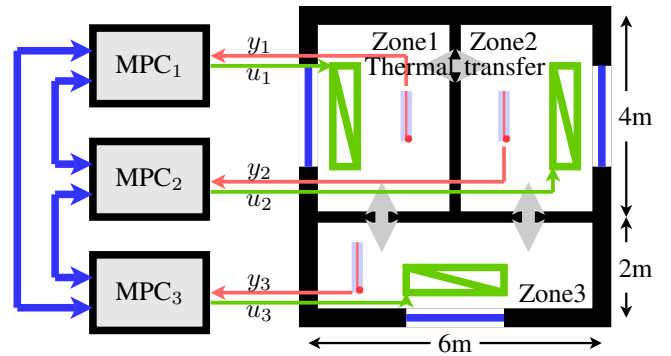


Fig. 4. Simulated building configuration

TABLE II  
PERFORMANCE COMPARISON

Control law	$I_C$ [ $^{\circ}Ch$ ]	$I_W$ [kWh]
On/Off	306	312
dMPC with $N_2$ fixed	285	284
dMPC with $N_2$ dynamic	283	276

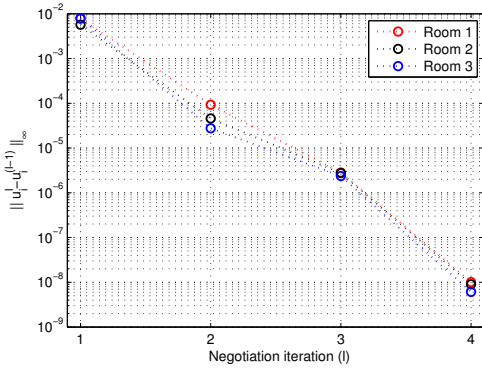


Fig. 5. Evolution of  $\|\mathbf{u}_i^{(l+1)}(k) - \mathbf{u}_i^l(k)\|_\infty$  for a given time step  $k$

Fig. 6 presents the results obtained simulating a one day period. The first part of the figure shows the reference tracking and the control inputs. The second part illustrates the prediction window sizes and the time evolution of the distributed and the decentralized parts of the exchanged output sequences  $\tilde{\mathbf{y}}_i(k)$ . Because of the slow dynamics of the thermal systems, the performance gain is not very significant minimizing local criterion over the iterations. The coupling variables,  $\tilde{\mathbf{y}}_j(k)$ , evolve very slowly between two consecutive optimizations which gives a fast convergence over negotiation steps, as shown in Fig. 5. Therefore, only one iteration ( $l_{max} = 1$ ) is sufficient in practice to calculate the control input, which gives a computational load similar to the decentralized approach. Note that the convergence test of the proposed algorithm can be made offline by computing the matrix  $\Theta$  for all the possible combinations of the set  $\{N_{2i}^c(k)\}$ ,  $i = 1, 2, \dots$ . In table II, the distributed MPC performance with fixed and dynamic prediction horizon is compared to a decentralized On/Off control law with an anticipation of  $N_{2i}^{max}T_s$  [s] of the occupation periods.

#### IV. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed an original control law to address the temperature regulation in large scale buildings. It is based on a distributed predictive control structure in which the idea is the use of a dynamic prediction horizon, depending on the occupation profile, allowing a better disturbance rejection. Firstly, this control strategy has been presented and tested for the temperature regulation of one room, then extended for a multi-zone building, resulting in a distributed control architecture, with a convergence analysis. The effectiveness of the proposed approach was illustrated by various simulations.

Regarding our future work, we plan to adapt this method to multiple heating sources case as well as taking into account the weather forecasts in the control algorithm. On a more theoretical way, we will try to formally guarantee the stability of the system controlled by the dynamic horizon dMPC.

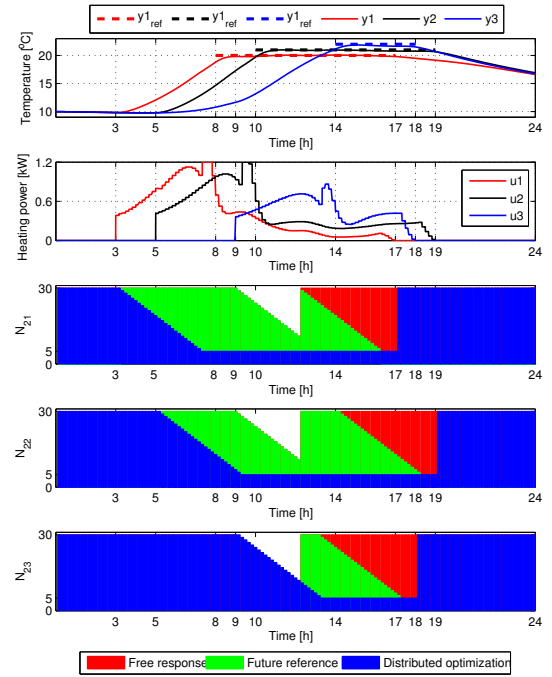


Fig. 6. Distributed predictive strategy behavior and the horizon evolution ( $\lambda_i = 1$ ,  $N_{ui} = N_{2i}^{min} = 5$ ,  $N_{2i}^{max} = 30$ ,  $i = 1..3$ ,  $T_s = 600s$ )

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