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Computer Simulation of an Optimal Stochastic Controller Applied to Passive Solar Rooms

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ABSTRACT

In rooms with significant solar gains, afternoon overheating is not a rare occurrence during sunny periods, especially if the heating system has a long response time. In order to solve this problem we have developed a predictive controller based on the theory of optimal stochastic control. This controller takes the anticipated solar gains into account which results in a lowered energy consumption and an improved thermal comfort.

Dynamic computer simulations have been used to determine the performance of the predictive controller on direct gain solar rooms with different heating systems. The results have been compared with simulations of classic controllers.

A 15% reduction of energy consumption has been obtained for the whole heating season. The reduction is 42% for certain months at the beginning and end of the heating season.

The development of the predictive controller, the systems studied and the results of the simulations are presented in this paper.

INTRODUCTION

In rooms with large solar gains, afternoon overheating is not a rare occurrence during sunny periods, especially if the heating system has a long response time. This is due to the fact that the thermal controller cannot take these random gains into account.

There are different classical methods to eliminate this overheating and the connected thermal discomfort for the inhabitants. These solutions are either to reduce the solar gains to a minimum by installing small or coated windows or to open the windows in the afternoon to let the heat out. Another solution, often found in the U.S., is to install an air-conditioning device. All these solutions will only partly exploit the solar gains in order to heat the rooms and the air-conditioning device will even use energy to cool the air.

Several attempts to develop deterministic controllers that use different types of weather forecast to account for solar gains have been made in recent years [1 - 4]. In this paper we explain in detail the development of a stochastic predictive controller based on the theory of optimal stochastic control [5 - 7] which use Markov chains [8] and AR-processes [9, 10] to model the solar radiation and outdoor temperature.

The performance of this controller compared to classic controllers has been evaluated with computer simulations. It has been tested on direct-gain rooms with different window openings and heating systems installed. The results of these simulations are also presented in this paper.

METHODS

Probabilistic models of perturbations

There are two different kinds of variables that influence the indoor temperature: meteorological variables like solar radiation, outdoor temperature, etc., and user-dependent variables like heat dissipated by lighting, electrical appliances, the human body, etc. Our study has so far only dealt with two meteorological variables, the solar radiation and the outdoor temperature.

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Different realizations of models for the solar radiation with Markov chains have been made [11 - 14]. We have used the model developed by Scartezzini, Rey and Liebling [11].

The model for the solar radiation consists of one daily and one hourly Markov chain. There are four different classes of "type of day" characterized by r corresponding to the ratio of the solar energy received on a horizontal surface I to the maximum energy possible I_0. Class one corresponds to a ratio of (0, 0.25), class 2 (0.25 - 0.5), etc. The hourly transmittance τ is divided into 10 different classes corresponding to the hourly solar radiation received on a horizontal surface FGH divided by the hourly maximum energy possible FGH_0. Class one corresponds to a ratio of (0, 0.1), class 2 (0.1, 0.2), etc. This is illustrated by Fig. 1.

The chains are based on one-month periods for the heating season October to April. Hourly data collected at Lausanne, Switzerland, (latitude 46°32′, altitude 420 m) by the Swiss Meteorological Institute for the winters 1978/79 until 1983/84 have been used to create the matrices.

The average temperature evolution $\overline{T}_{ej}(t)$ for each type of day and each month has been calculated using the SMI data (see Fig. 1).

Synthetic data has been generated and compared with measured data in order to test the validity of the model. Figure 2 describes how synthetic data is generated and Fig. 3 shows the histograms for the two sets of data. A X^2 test has been performed to determine that the two distributions are the same at a significance level of 5%.

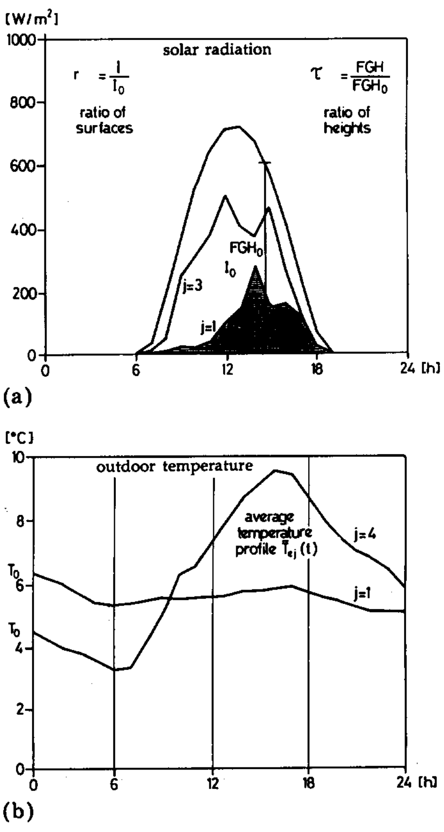

Fig. 1. Model of solar radiation and air temperature. (a) $r \in [(j-1) \times 0.25, j \times 0.25]$ for $j = 1,...,4$. $\tau \in [(k-1) \times 0.10, k \times 0.10]$ for $k = 1,...,10$.

Control algorithm

The control algorithm used in this work is based on the theory of optimal stochastic control [5 - 7]. According to this theory, the cost function J is defined by

$$J(t_0) = \sum_{i=1}^{N} \{C_1 U(t_0 + i\Delta t) + C_2[\exp(\mathrm{PMV}^2(t_0 + i\Delta t)) - 1]\} \quad (1)$$

the expected value of J is minimized over the horizon N, according to the probabilities of transition for the Markov chains defined above. This function is expressed by two terms, the first is dependent on the energy consumption U, and the second is dependent on the comfort expressed by the PMV (predicted mean vote) according to the theory of Fanger [15]. The two coefficients C_1 and C_2 allow a relative weighting of the two terms. They have been chosen so that maximum heating during one hour corresponds to a PMV of ±0.2. This allows for

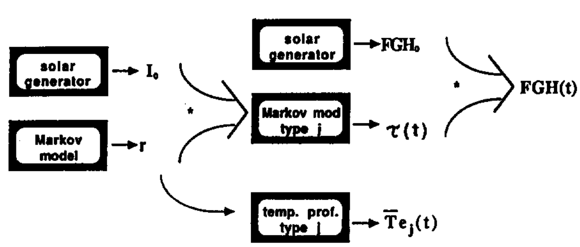

Fig. 2. Generation of synthetic data using the probabilistic weather model. Markov chains generate type of day j and hourly atmospheric transmission $\tau(t)$.

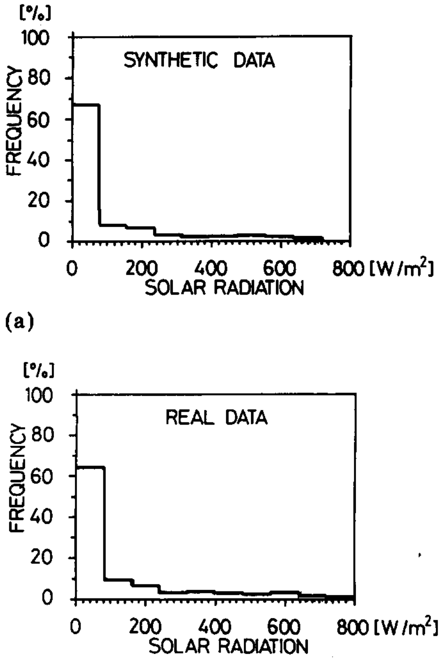

Fig. 3. Histograms of (a) synthetic and (b) real data of horizontal solar radiation (Lausanne, 46°32′ N, March).

6% of the occupants to be dissatisfied with the thermal comfort. At optimal comfort (PMV = 0) there are 5% of the occupants that are dissatisfied.

The dynamic behaviour of the solar rooms is described by a 5-node thermal model characterized by a set of linear equations,

$$T(t + \Delta t) = \mathbf{A}T(t) + \mathbf{B}Y(t) + \mathbf{C}U(t) \qquad (2)$$

This model in eqn. (2) is integrated into the controller in order to account for the thermal characteristics of the considered solar system. The temperature of the nodes is represented by the vector T, the meteorological variables by the vector Y, and U characterizes the command given to the system. U represents the auxiliary energy supplied to the system. The matrixes **A**, **B** and **C** are functions of the thermal properties of the considered room.

The optimal command for each possible state of the vectors T, Y and U is calculated according to the theory of optimal stochastic control at each time step over the horizon N. The indoor air temperature has been divided into 10 intervals from 15 °C to 24 °C and the temperature of the massive construction (walls, ceiling, floor) has been divided into 15 intervals ranging from −1.2 °C to +1.8 °C of the massive temperature at midnight. The solar radiation is divided into 10 intervals according to the model described above and the outdoor temperature evolution is considered to be deterministic and equal to the average temperature profile shifted with the temperature at midnight. The horizon is 24 hours with one-hour time steps. There are five possible commands ranging from no to maximum heating.

The same cost function eqn. (1) and room evolution eqn. (2) have been used to determine the maximum performance of the predictive controller when the actual weather evolution is known in advance. This controller will be referred to as exact forecast controller to distinguish it from the predictive controller based on the theory of optimal stochastic control.

Classic controllers

Three different types of classic controllers have also been simulated in order to give reference points: they are an outdoor temperature regulator with thermostatic valves, an ideal indoor air thermostat, and a comfort thermostat.

The most common control technique today in Switzerland is the outdoor temperature controller. It calculates how much energy it is necessary to supply to the system according to the difference in outdoor temperature T_e (°C) and the desired indoor air temperature T_{set} (°C), accounting for the specific losses of the room P (W/K). A cut-off point at $T_e = 12$ °C is also simulated as well as thermostatic valves which intervene to cut off the energy supply if the indoor-air temperature exceeds T_{set}.

The ideal indoor-air thermostat calculates at each moment how much energy it is necessary to supply to the room in order to keep the air temperature at T_{set}. In other words, it is a thermostat without hysteresis.

The comfort thermostat is not really a classic regulator but it has been introduced in order to distinguish between the effects of the forecast and the structure of the cost function. The comfort thermostat uses a measurement of PMV based on the theory of Fanger [15].

All regulators (classic and predictive) have been set so that they supply an air tempera ture or index temperature of 20 °C during the day (06:00 - 22:00) and 18 °C overnight (22:00 - 06.00).

Studied systems

Two direct-gain test rooms, with different size windows on their south façade, have been simulated. They are identical to the test rooms studied during the work of IEA Annex XII [16]. The first room has a window opening area which is 20% of the floor surface and the window opening area of the second is 30%. Figure 4 shows the first room and gives its characteristics, the figures in parenthesis represent the second room.

The rooms are situated in the corner of a building. The east- and south-facing walls of the rooms are external, and the south-facing wall contains the glazing.

Two different types of heating system have been considered; one air convector with instant response time, and one floor heating system with a response time of four hours. The heating systems are also represented in Fig. 4. The air convector-heated system is dimensioned at 1500 W and the floor-heated system at 2000 W. This difference is necessary in order to assure the same comfort in the two cases. It is due to the large heat losses through the floor to the room situated below.

These rooms are described with a 33-node thermal model that takes heat transfers via conduction, convection and infrared radiation into account. This model is introduced into the simulation program PASSIM [17] which has had the control algorithm incorporated.

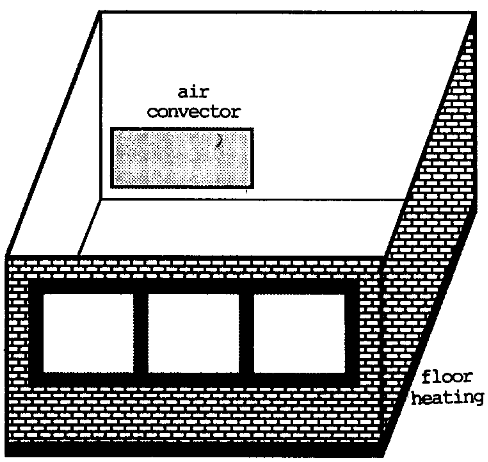

Fig. 4. The simulated room with 20% window opening area. The two simulated heating systems have the following values: those in parenthesis correspond to the room with the larger windows.

Volume	75 m^3	(75)
Glazing	4.9 m^2	(7.2)
Floor	30 m^2	(30)
Loss coeff.	30 W/K	(36)
Capacity	15 MJ/K	(15)
Response time: Floor heating	4 h	
Response time: Air Convector	0 s	

RESULTS

Energy performance

The energy consumption has been determined for all systems defined above for the heating season of 1984/85.

These values are presented in Table 1. The numbers in the columns (%) give the percentage of energy savings compared to a controller based on the outdoor temperature. The column "Exact forecast" corresponds to the energy consumption obtained when the cost function is minimized using the exact knowledge of the weather evolution. The floor heating systems lose a large part of their energy to the room situated below and the lost energy is used to heat this room. This explains the larger energy consumption for the floor-heated systems. In a floor-heated building with many levels, the heat losses will be compensated by heat gains from the room above. The total energy consumption for the building will be comparable to that with an air-convector heating system.

The larger energy consumption for the month of January is due to extreme outdoor temperatures (−20 °C for three weeks).

The energy consumption for the predictive controller is smaller than that for the conventional controllers. The energy saving compared to a controller based on the outdoor temperature varies between 5.9% and 15.3%. The saving for the exact forecast controller is between 10.5% and 21.5%. The energy savings are larger for the rooms with large window openings as well as for the rooms with large inertia systems. As expected, the energy saving is larger for systems where the factor of utilization of solar gains is small. There is a larger potential of energy saving for these systems which is exploited by the predictive controller.

TABLE 1

The energy consumption for the winter 1984/85 for the studied systems

System	Type of regulation									
	Outdoor temperature		Indoor air thermostat		Comfort thermostat		Predictive control		Exact forecast	
	(MJ)	(%)	(MJ)	(%)	(MJ)	(%)	(MJ)	(%)	(MJ)	(%)
Opening 30%										
Floor heating	8435	—	8054	4.5	7680	9.0	7143	15.3	6622	21.5
Air convector	4440	—	4071	8.3	4025	9.3	4009	9.7	3680	17.1
Opening 20%										
Floor heating	7703	—	7395	4.0	7291	5.4	6926	10.1	6438	16.4
Air convector	4086	—	3803	6.9	3980	2.6	3846	5.9	3658	10.5

The comparison between the predictive controller and the comfort thermostat verifies that a large fraction of the energy savings is due to the prediction and not only to the form of the cost function. Figure 5 shows the monthly energy consumption for the floor-heated room with the larger window opening area (30%).

The reduction in energy consumption is larger for the sunny months at the beginning and the end of the heating period. It is equal to 42% for October 1984.

Thermal comfort

The comfort for the inhabitants in the different rooms has also been determined according to the theory of Fanger [15]. The distributions of PMV (predicted mean vote) are shown in Fig. 6 for the month of November, 1984, in the floor-heated room with the window opening of 30%. The shape of these distributions, as well as the average value, shows the large increase in thermal comfort obtained by the predictive controller. It should be noted that this excellent result was obtained for a heating system (floor heating) which is normally incompatible with direct-gain solar systems (overheating and small use of solar gains). For the air convector-heated rooms the reduction in energy consumption was obtained with a comfort comparable to that of an indoor air thermostat.

DISCUSSION

The largest problem with implementation of optimal stochastic control up to now has

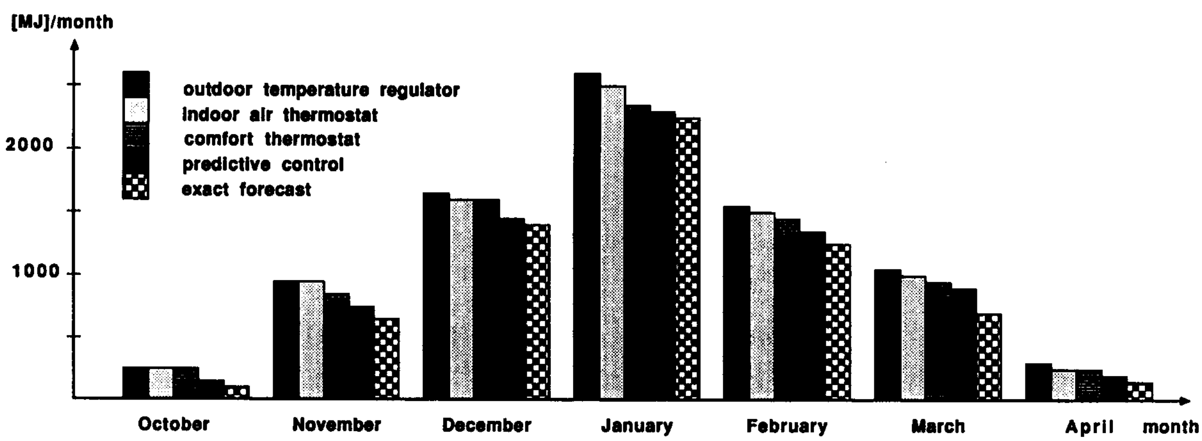

Fig. 5. Monthly energy consumption for the heating season 1984/85, floor heating and 30% window opening area.

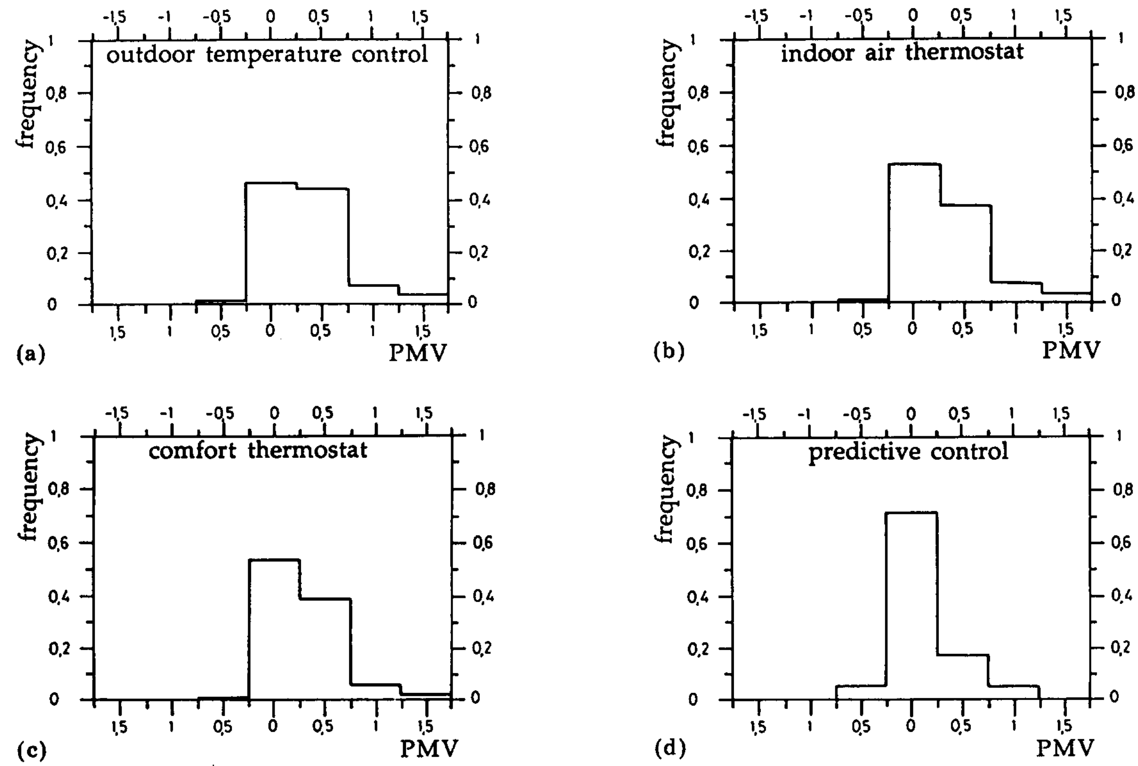

Fig. 6. Distribution of the PMV obtained for different types of control (November 1984, floor heating, window opening 30%). (a) outdoor temperature, $\overline{PMV} = 0.35$, $Q_{aux} = 969$ MJ. (b) indoor air thermostat, $\overline{PMV} = 0.33$, $Q_{aux} = 931$ MJ. (c) comfort thermostat, $\overline{PMV} = 0.27$, $Q_{aux} = 872$ MJ. (d) predictive control, $\overline{PMV} = 0.09$, $Q_{aux} = 770$ MJ.

been the need for large memories and long calculation times in order to determine the matrix of commands.

The memory space necessary to implement this control algorithm is 2.8 Mbytes and the calculation time is six minutes CPU on VAX 8530. Once the matrix has been calculated at midnight, the control algorithm is very fast and it consists only of a pointer that will read the optimal command within the matrix.

In the continuation of this work we will look at different ways to reduce the demand on the computer. A parametric study, to optimize the horizon and the number of possible commands and temperatures, will be performed.

An experiment will also be performed where the predictive controller is compared to a conventional controller. The need for memory and calculation time is not excessive to implement on a PC that can control the heating system.

CONCLUSIONS

The principle of a predictive controller based on the theory of optimal stochastic control has been presented. Dynamic computer simulations have been used in order to determine the performance of this controller compared to conventional controllers for direct-gain solar rooms with different window opening areas and different heating systems [18].

The reduction in energy consumption compared to a controller based on the outdoor temperature is a function of the window opening area and of the inertia of the heating system. For the studied cases (20% and 30% window opening area) the reduction in energy consumption is 10% and 15% for the floor-heated room, and 6% and 10% for the room heated with air convectors.

The comfort is significantly improved for the rooms with floor heating when using the predictive controller. When the rooms are heated with air convectors, the reduction in energy consumption is obtained for the same level of comfort.

The extension of this work is to install a predictive controller in an experimentally set-up room and to compare the performances with another room using a conventional controller. If the experiment confirms the results presented here, it is probable that these controllers will be used in practice to resolve the problems encountered with floor heating in buildings with large solar gains.

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