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Model predictive control of a building heating system: The first experience

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ABSTRACT

This paper presents model predictive controller (MPC) applied to the temperature control of real building. Conventional control strategies of a building heating system such as weather-compensated control cannot make use of the energy supplied to a building (e.g. solar gain in case of sunny day). Moreover dropout of outside temperature can lead to underheating of a building. Presented predictive controller uses both weather forecast and thermal model of a building to inside temperature control. By this, it can utilize thermal capacity of a building and minimize energy consumption. It can also maintain inside temperature at desired level independent of outside weather conditions. Nevertheless, proper identification of the building model is crucial. The models of multiple input multiple output systems (MIMO) can be identified by means of subspace methods. Oftentimes, the measured data used for identification are not satisfactory and need special treatment. During the 2009/2010 heating season, the controller was tested on a large university building and achieved savings of 17–24% compared to the present controller.

1. Introduction

According to the U.S. Energy Information Administration, in 2005, buildings accounted for 39% of total energy usage, 12% of the total water consumption, 68% of total electricity consumption, and 38% of the carbon dioxide emissions in the U.S.A. [1]. Although the energy efficiency of systems and components for heating, ventilating, and air conditioning (HVAC) has improved considerably over recent years, there is still potential for substantial improvements. This article deals with an advanced control technique, that can provide significant energy savings in comparison with conventional, non-predictive techniques.

Widely used control strategy of water heating systems is the weather-compensated control. This feedforward control can lead to poor energy management or reduced thermal comfort even if properly set up, because it utilizes current outside temperatures only. Weather conditions, however, can change dramatically in few hours; and due to the heat accumulation in large buildings, it can lead to underheating or overheating of the building easily.

During recent years, significant advances have been done for the HVAC control systems [2–6]. For instance, continuous adaptation of control parameters, optimal start–stop algorithms, optimization of energy loads shifting [7], or inclusion of free heat gains in the control algorithm are particular improvements of the building heating system [8]. Some new concepts have been verified by simulations [9,10], nevertheless they are still waiting for real operations. The model predictive control, [11–15] (MPC) presented in this article introduces a different approach to the heating system control design. As the outside temperature is one of the most influential quantity for the building heating system, weather forecast is employed in the predictive controller. It enables to predict inside temperature trends according to the selected control strategy. The aims of the control can be expressed in natural form as thermal comfort and economy trade off. Unfortunately, this concept has some drawbacks, such as extensive computational requirements or necessity of a mathematical model of the physical system (building in this case).

All these issues are discussed in detail in following sections, which are organized as follows. Section 2 compares the current control techniques with MPC. Section 3 introduces model predictive control concept more in detail and explains the mathematical background of this technique. This section also addresses new modified zone model predictive controller. Problem of the model identification is discussed as well. Application results are summarized in Section 4. Remarks to future development are outlined in Section 5. The last section concludes the work.

List of abbreviations used throughout the article is mentioned in Table 1.

2. Current heating control strategies

Let us briefly compare the major state-of-the-art heating control techniques – on–off room temperature control, weather-

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Table 1
Notation.

Abbrev.	Meaning
ARX	Auto-regressive model with external inputs
ARMAX	Auto-regressive, moving average model with external inputs
CTU	Czech Technical University in Prague
HVAC	Heating, ventilation and air-conditioning systems
MIMO	Multiple-input, multiple-output systems
MPC	Model predictive control
OE	Output error model
PID	Proportional – integrative – derivative controller
SISO	Single-input, single-output systems
WC	Weather-compensated control

compensated control, and PID [8] control – with the proposed application of MPC.

The *on–off room temperature control* is the simplest type of control; the heating devices in a room are switched on and off (device state S) according to some room temperature error ($e_t = t_{\text{required}} - t_{\text{room}}$) threshold, usually implemented as a suitable hysteresis curve $f_{\text{on-off}}$:

$$S = f_{\text{on-off}}(e_t) \tag{1}$$

This is a very simple feedback control, which does not contain any information about the dynamics of the building. The main advantage is its simplicity.

On the contrary, the *weather-compensated control* is a feedforward control, which also does not contain any information about the building dynamics. The temperature of the heating medium, such as water (t_{water}), is set according to the outside temperature t_{outside} by means of predetermined heating curves $f_{\text{w-c}}$, that is

$$t_{\text{water}} = f_{\text{w-c}}(t_{\text{outside}}) \tag{2}$$

In spite of the lack of dynamics in the control, this is a long used and proven control strategy; its advantage is its robustness and simple tuning.

PID control is one of the most favorite strategies of control engineers [16,17]. It is a feedback control with some information about the system dynamics, that is, the heating water temperature t_{water} is determined according to the room temperature error e_t and "some" history:

$$t_{\text{water}} = f_{\text{PID}}(e_t, \text{history}) \tag{3}$$

PID controllers are robust and allow accurate tuning, but they cannot reflect the outside temperature effects. This is the reason why PIDs in HVAC control are not as common as in other control applications.

Even though all the above controllers are easy to tune for single-input, single-output (SISO) systems, their tuning for multiple-input multiple-output (MIMO, sometimes called multidimensional) systems becomes very difficult or even impossible in practice. The PID control can be applied to MIMO systems only in very rare occasions, in case of specially structured (input–output decoupled) systems.

We would therefore appreciate some control strategy, which would have a feedback (i.e. the room temperature error e_t is used), use as much information as possible (the outside temperature t_{outside}, the weather forecast $t_{\text{predicted}}$, and others x) and include some system dynamics ("history") as well. This can be formalized – in the spirit of the above Eqs. (1)–(3) – as

$$t_{\text{water}} = f_{\text{MPC}}(e_t, t_{\text{outside}}, t_{\text{predicted}}, x, \text{history}) \tag{4}$$

These requirements are satisfied by a so-called model (based) predictive controller (MPC), which is specially suitable for systems with multiple inputs and multiple outputs, which is very typical for heating systems. Its main drawbacks are high demands for computational resources and non-trivial mathematical background, especially in the "Model" part of the controller.

Fig. 1. The building of the Czech Technical University in Prague that was used for MPC application.

3. Model predictive control

3.1. State of the art

Model (based) predictive control (MPC) is a method of advanced control originated in late seventies and early eighties in the process industries (oil refineries, chemical plants, etc.) [11]. The MPC is not a single strategy, but a vast class of control methods with the model of the process explicitly expressed trying to obtain control signal by minimizing objective function subject to (in general) some constraints [18]. The minimization is performed in an iterative manner on some finite optimization horizon to acquire N step ahead prediction of control signal that leads to minimum criterion subject to all constraints. This, however, carries lots of drawbacks such as no feedback, no robustness, and no stability guarantee. Many of these drawbacks can be overcome by applying so-called receding horizon, i.e. at each iteration only the first step of the control strategy is implemented and the control signal is calculated again, thus, in fact, the prediction horizon keeps being shifted forward. Stability of the constrained receding horizon has been discussed in Refs. [13,14], or yet another approach using robust control design approach [15].

There were several attempts made to utilize predictive control concept in HVAC in the last decade [19,9,20,21,10]. Complex view into area of optimal building control gives the project OptiControl.[1] Besides its own results, it also provides a wide range of references to the related articles. Another project worth to mention is the predictive networked building control that deals with predictive control of the thermal energy storage on the campus of the UC-Berkeley.[2] Most of the articles devoted to the HVAC predictive control conclude results just by numerical simulations. On the contrary, this article describes MPC being tested on the real eight-floor building (see Fig. 1).

3.2. Principles

We will now briefly describe the basic ideas lying behind the MPC. To be more illustrative, we will take the course of the MPC implementation in our own project; even though the experienced practitioners in heating control are rather conservative in their field, they can accept new method, such as MPC, if performed in small, consecutive steps, which helps them to get acquainted with its principles.

[1] http://www.opticontrol.ethz.ch.
[2] http://sites.google.com/site/mpclaboratory/research/predictive-networked-building-control-1.

Having a well working control, such as weather-compensated control of a building, it is often unwise to change it to something novel, but unproven. However, it can be very advantageous to provide a "tool" that would enhance the possibilities of the existing system. A mathematical model can be such a "tool", allowing the system operators to predict the behavior of the building. If the model is accurate enough (e.g. as a one-day predictor), another feature can be added—the operator can experiment with the model and try some "what if" scenarios. The next step is obviously implementation of an algorithm that proposes the best scenarios; it is still a "tool", the model and algorithm are not involved in the control loop. That would be the last step – after the operator begins to trust the algorithm, he begins to ask for the closer of the control loop incorporating what we call model predictive control.

To be more precise, the first step is to find a dynamic model P

$$y = P(u, t) \tag{5}$$

where y is the output, u is the input (both can be vectors) and t is time. Inputs u may be entered by the operator in the beginning, such that he can see the expected behavior of the system, as seen on outputs y. The next step is finding the optimal inputs u automatically. This can be achieved by introducing an optimality criterion $J(y, u, t)$, wherein the control demands are described in the language of mathematics. Substituting from (5), the optimal control inputs can be found by computing

$$u_{\text{optimal}} = \min_{u} J(P(u, t), u, t) \tag{6}$$

subject to "some" constraints. This very basic idea will now be discussed more in detail.

3.3. Model identification

One of the crucial contributors to the quality of the control is a well identified model which will be later on used for control in MPC algorithm. There are several completely different approaches to system identification (see e.g. [22,23]). Some of them use knowledge of system physics, while others exploit statistical data, such as grey-box [24,25] (some prior information such as system structure is known in advance) or black-box identification. Grey box methods using models such as ARX, ARMAX, OE and others are well established among the practitioners as well as theoreticians. There is, however, a significant problem, when multiple input multiple output (MIMO) systems are considered. The standard input–output identification methods are not capable of dealing with such a model, thus one has to either reformulate the problem to several single-output cases, or to use state-space identification methods, such as subspace methods. The first approach, including computer modeling of the building, as well as comparison of ARMAX model and subspace methods, was briefly described in [26].

The main difference between classical and subspace identification can be seen in Fig. 2 (see Ref. [27]). Given the sequence of input and output data, $u(k)$ and $y(k)$, respectively, do:

- **Classical approach**. Find system matrices, then estimate the system states, which often leads to high order models that have to be reduced thereafter.
- **Subspace approach**. Use orthogonal and oblique projections to find Kalman state sequence, then obtain the system matrices using least squares method. Here we introduce the latter—subspace identification methods.

The objective of the subspace algorithm is to find a linear, time invariant, discrete time model in an innovative form:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ke(k) \\ y(k) &= Cx(k) + Du(k) + e(k), \end{aligned} \tag{7}$$

based on given measurements of the input $u(k) \in \mathbb{R}^m$ and the output $y(k) \in \mathbb{R}^l$ generated by an unknown stochastic system of order n, which is equivalent to the well-known stochastic model:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w(k) \\ y(k) &= Cx(k) + Du(k) + v(k), \end{aligned} \tag{8}$$

with covariance matrices Q, S and R of process and measurement noise sequences as follows:

$$\operatorname{cov}(w, v) = E\left(\begin{bmatrix} w(p) \\ v(p) \end{bmatrix} \begin{bmatrix} w^T(q) & v^T(q) \end{bmatrix} \right) = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \delta_{pq} \geq 0, \tag{9}$$

and with A, B, C, and D denoting system matrices and K and e in (7) is Kalman gain – derived from the Algebraic Riccati Equation (ARE) [28], and white noise sequence, respectively. Loosely speaking, the objective of the algorithm is to determine the system order n and to find the matrices A, B, C, D and K.

3.3.1. Data matrices for subspace algorithm

The following matrices are necessary to form for subspace algorithm. Notation was adapted as in Ref. [27]. Upper index d denotes deterministic subsystem, while the upper index s denotes stochastic subsystem. Two kinds of matrices are used for subspace algorithm, data and system related matrices.

- **Data matrices**. Input and output block Hankel matrix are formed from input and output data as follows:

$$U_{0|2i-1} = \left(\begin{array}{ccccc} u_0 & u_1 & u_2 & \cdots & u_{j-1} \\ u_1 & u_2 & u_3 & \cdots & u_j \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{i-1} & u_i & u_{i+1} & \cdots & u_{i+j-2} \\ \hline u_i & u_{i+1} & u_{i+2} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & u_{i+3} & \cdots & u_{i+j} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{2i-1} & u_{2i} & u_{2i+1} & \cdots & u_{2i+j-2} \end{array} \right) \quad Y_{0|2i-1} = \left(\begin{array}{ccccc} y_0 & y_1 & y_2 & \cdots & y_{j-1} \\ y_1 & y_2 & y_3 & \cdots & y_j \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{i-1} & y_i & y_{i+1} & \cdots & y_{i+j-2} \\ \hline y_i & y_{i+1} & y_{i+2} & \cdots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & y_{i+3} & \cdots & y_{i+j} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{2i-1} & y_{2i} & y_{2i+1} & \cdots & y_{2i+j-2} \end{array} \right), \tag{10}$$

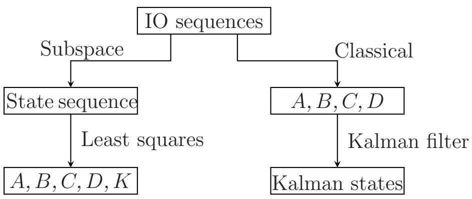

Fig. 2. Comparison between classical and subspace identification methods.

which can be written in shorten form as follows:

$$\begin{pmatrix} \frac{U_{0|i-1}}{U_{i|2i-1}} \end{pmatrix} = \begin{pmatrix} \frac{U_p}{U_f} \end{pmatrix}$$
$$\begin{pmatrix} \frac{Y_{0|i-1}}{Y_{i|2i-1}} \end{pmatrix} = \begin{pmatrix} \frac{Y_p}{Y_f} \end{pmatrix}, \tag{11}$$

where matrices U_p and U_f represent past and future inputs, respectively. Outputs $y(k)$ and noise $e(k)$ related matrices can be formed in similar manner. Grouped data matrix consisting of past input and past output data is formed as follows:

$$W_p = W_{0|i-1} = \begin{pmatrix} \frac{U_{0|i-1}}{Y_{0|i-1}} \end{pmatrix}.$$

- **System related matrices**. Extended ($i > n$) observability (Γ_i) and reversed extended controllability (Δ_i) matrices for deterministic and stochastic subsystems, respectively are defined as follows:

$$\Gamma_i = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{pmatrix} \tag{12}$$

$$\Delta_i^d = \begin{pmatrix} A^{i-1}B & A^{i-2}B & \dots AB & B \end{pmatrix} \tag{13}$$

$$\Delta_i^s = \begin{pmatrix} A^{i-1}K & A^{i-2}K & \dots AK & K \end{pmatrix} \tag{14}$$

Algorithm also uses lower block triangular Toeplitz matrix for deterministic and stochastic subsystem, respectively:

$$H_i^d = \begin{pmatrix} D & 0 & 0 & \dots & 0 \\ CB & D & 0 & \dots & 0 \\ CAB & CB & D & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{i-2}B & CA^{i-3}B & CA^{i-4}B & \dots & D \end{pmatrix}$$

$$H_i^s = \begin{pmatrix} I & 0 & 0 & \dots & 0 \\ CK & I & 0 & \dots & 0 \\ CAK & CK & I & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{i-2}K & CA^{i-3}K & CA^{i-4}K & \dots & I \end{pmatrix}. \tag{15}$$

3.3.2. Subspace algorithm

The entry point to the algorithm are input–output equations as follows:

$$\begin{aligned} Y_p &= \Gamma_i X_p + H_i^d U_p + H_i^s E_p \\ Y_f &= \Gamma_i X_f + H_i^d U_f + H_i^s E_f \\ X_f &= A^i X_p + \Delta_i^d U_p + \Delta_i^s E_p. \end{aligned} \tag{16}$$

Oblique projection as described in Refs. [29,27] is the main tool used in subspace methods is defined as follows:

$$\mathcal{O}_i = Y_f \underset{U_f}{/} W_p. \tag{17}$$

The order of the system can be determined from analysis of singular values obtained using singular value decomposition (SVD) of $W_1 \mathcal{O}_i W_2$, where W_i are weighting matrices of particular size and determine resulting state space basis. It has been shown [27], that $\mathcal{O}_i = \Gamma_i \tilde{X}_i$, where $\tilde{X}_i$ is Kalman filter state sequence. This factorization also yields extended observability matrix Γ_i and Kalman filter states $\tilde{X}_i$.

Algorithm continues from either Γ_i or $\tilde{X}_i$ in a slightly different manner depending on particular subspace identification algorithm, however, both ways lead to a computation of system matrices A and C using least squares method.

Computation of system matrices B and D follows subject to matrices A and C computed in previous step. Different approaches for matrices determination are addressed in detail in Ref. [27].

The algorithm concludes with computation of Kalman gain matrix K in a standard way using state and output noise covariance matrices (9) which are computed from residuals of the previous computations.

The model structure used in MPC is the state-space model (7) identified by subspace identification (described in Section 3.3) from measured data. The application of the model will become apparent later in this section.

3.4. Predictive controller

3.4.1. MPC strategy

The MPC strategy comprises two basic steps:

- The future outputs are predicted in an open-loop manner using the model provided information about past inputs, outputs and future signals, which are to be calculated. The future control signals are calculated by optimizing the objective function, i.e. chosen criterion, which is usually in the form of quadratic function. The criterion constituents can be as follows:
  - errors between the predicted signal and the reference trajectory $y_r(k)$;
  - control effort;
  - rate of change in control signals.
- The first component of the optimal control sequence $u(k)$ is sent to the system, whilst the rest of the sequence is disposed. At the next time instant, new output $y(k+1)$ is measured and the control sequence is recalculated, first component $u(k+1)$ is applied to the system and the rest is disposed. This principle is repeated *ad infinitum* (receding horizon).

The reference trajectory $y_r(k)$, room temperature in our case, is known prior, as a schedule. The major advantage of MPC is the ability of computing the outputs $y(k)$ and corresponding input signals $u(k)$ in advance, that is, it is possible to avoid sudden changes in control signal and undesired effects of delays in system response.

Standard formulation of criterion for MPC is as follows:

$$J = \sum_{k=0}^{N-1} q(k)(y(k) - y_r(k))^2 + r(k)u(k)^2, \tag{18}$$

where $q(k)$ is weight for difference between system output $y(k)$ and reference $y_r(k)$ at time instant k, while $r(k)$ is the weight of the displacement of control signal $u(k)$. If the future desired output value is known in advance, then this criterion leads to such an optimal system input, which minimizes weighted square of $y(k) - y_r(k)$. By

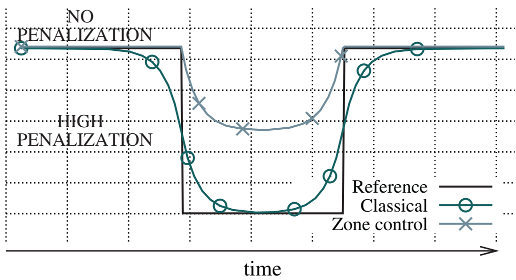

Fig. 3. Comparison between classical and zone predictive strategy. Weighting of entirely negative errors makes predictive controller to follow accurately the upper part of reference trajectory. When step down of desired value occurs, the system output drops to the reference value with a minimum control effort.

this, the area delimited by the system output below desired value is same as the area above it. This is depicted in Fig. 3 by line marked with circles. Such a behavior is satisfactory for most of the common control problems but not for temperature control of a building. The aim of the control is to adhere the upper desired value from its beginning to its end. Resulting behavior of the output is delineated in Fig. 3 by line with crosses.

This unusual problem can be solved by several approaches:

- The intuitive method is to use dynamic weights $q(k)$ and $r(k)$ at time, i.e. to make them time-dependant. These weights then depend on the shape of the reference trajectory – if there is a step-up/down on a prediction horizon, then weight $q(k)$ is set to be greater than $r(k)$ for k when the reference trajectory is on upper level, whilst $q(k) < r(k)$ for the rest of k on prediction horizon. This simple procedure ends if there exists more reference trajectory levels than two (but in this case is the best way how to solve such a problem).
- The second approach is as follows: In the minimization of the criterion (18) the reference trajectory y_r can be substituted with “artificial” reference w, which can be some approximation from the actual output y to real reference y_r. This can be done using following convex combination [30]:

$$\begin{aligned} w(k) &= y(k) \\ w(k+i) &= \alpha w(k+i-1) + (1-\alpha) y_r(k+i), \end{aligned} \tag{19}$$

  where $i = 1 \ldots N$ and $\alpha \in \langle 0;1 \rangle$ is a parameter, that determines the smoothness (and speed) of the approaching of the real output to the real reference. (19) can be also restated as follows:

$$\begin{aligned} w(k) &= y(k) \\ w(k+i) &= \alpha r(k+i) - \alpha^i (y(k) - y_r(k)). \end{aligned} \tag{20}$$

  Making use of artificial reference may be very helpful in the case of number of “steps” in reference trajectory with need of its precise tracking by the actual output.
- Completely different way is to reformulate the part of criterion (18), which refer to the desired value error. If $y(k) < y_r(k)$ then weight the square of this difference using $q(k)$, otherwise the error is not weighted. This can be treaded by using the concept of zone control (also called funnel MPC) [18] where the reference error is not weighted in a specified interval while the weighting out is made in a common way. The lower bound of the interval is in our case desired value, whilst the upper bound is not specified due to the fact, that the building naturally tends to underheat providing the weighted output. Such a method can be used for tracking of reference trajectory with arbitrary number of levels.

The last approach will be discussed in detail.

3.4.2. MPC problem formulation

For a given linear, time invariant, discrete-time deterministic model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{21}$$

find the optimal control sequence on the horizon of prediction (length N) by minimizing the objective function

$$J = \sum_{k=0}^{N-1} q(k)(y(k) - z(k))^2 + r(k)u(k)^2, \tag{22}$$

subject to

$$\begin{aligned} u_{\min} \leq \quad & u(k) \quad \leq u_{\max} \\ y_r(k) \leq \quad & z(k) \\ \Delta_{\max} \geq \quad & |u(k) - u(k-1)| \end{aligned} \tag{23}$$

where constraints $u_{\min}$, $u_{\max}$ are the minimum and maximum values of the control signal, $y_r(k)$ is desired value, thus lower bound for $z(k)$ and $\Delta_{\max}$ is a maximum rate of change of the control signal.

The objective function J (in (22)) can be rewritten into a matrix form (denoted without specification of a time instant)

$$J = (y - z)^T Q (y - z) + u^T R u, \tag{24}$$

where Q and R are weighting matrices of output error and control effort, respectively. The trajectory of the output is given as:

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{N-1} \end{bmatrix} x(0) + \begin{bmatrix} D & & & \\ CB & D & & \\ \vdots & & \ddots & \\ CA^{N-2}B & \ldots & CB & D \end{bmatrix} \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix}, \tag{25}$$

i.e.

$$y = \Gamma x(0) + Hu, \tag{26}$$

where Γ is extended observability matrix and H is a matrix of impulse responses. Let $\tilde{y} = \Gamma x(0)$, then using (26), we can rewrite (24) as follows:

$$J = (\tilde{y} + Hu - z)^T Q (\tilde{y} + Hu - z) + u^T R u. \tag{27}$$

Minimization of such an objective function is a nonlinear programming problem, which can be readily rewritten into quadratic programming problem

$$\begin{aligned} J = & \begin{bmatrix} u^T & z^T \end{bmatrix} \begin{bmatrix} H^T QH + R & -H^T Q \\ -QH & Q \end{bmatrix} \begin{bmatrix} u \\ z \end{bmatrix} + \\ & +2 \begin{bmatrix} \tilde{y}^T QH & -\tilde{y}^T Q \end{bmatrix} \begin{bmatrix} u \\ z \end{bmatrix} + \tilde{y}^T Q \tilde{y} \end{aligned} \tag{28}$$

This yields the optimization problem $\min_{u,z} J$, which can be effectively solved using some of the computer algebra systems. The resulting problem has $(m+p) \cdot T$ variables which is a greater dimension compared to the classical one (described by criterion (18)) with $m \cdot T$ variables, where m and p are number of inputs and outputs respectively.

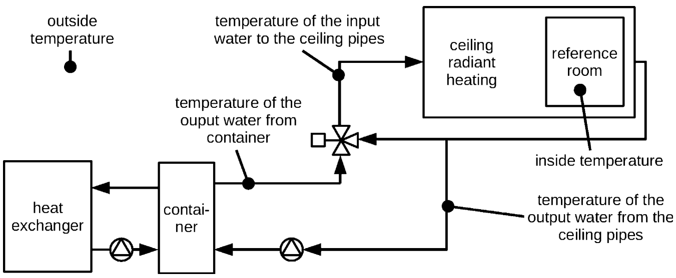

Fig. 4. Simplified scheme of the ceiling radiant heating system.

4. Application

The methods described in the previous sections were tested through December 2009 and January 2010 and the the real run of control application using proposed control strategy started in February 2010 at the building of the Czech Technical University in Prague. As of February 2010 the whole building consisting of 7 control blocks is controlled by presented MPC algorithm. All algorithms were implemented in Scilab.[3]

4.1. Description of the building

The building of the Czech Technical University in Prague uses a "Crittall" type ceiling radiant heating and cooling system. The "Crittall" system, invented in 1927 by R.G. Crittall and J.L. Musgrave [31], was a favorite heating system in the Czech Republic during 1960s for large buildings. In this system, the heating (or cooling) beams are embedded into the concrete ceiling. The control of individual rooms is very complicated due to the technical state of the control elements in all rooms. The control is therefore carried out for one entire building block, i.e. the same control effort is applied to all rooms of the building block. There are two (out of seven control blocks) building blocks with the same construction and orientation. Therefore, this situation is ideal for comparison of different control strategies, as depicted in Fig. 5.

A simplified scheme of the ceiling radiant heating system is illustrated in Fig. 4. The source of heat is a vapor–liquid heat exchanger, which supplies the heating water to the water container. A mixing occurs here, and the water is supplied to the respective heating circuits. An accurate temperature control of the heating water for respective circuits is achieved by a three-port valve with a servo drive. The heating water is then supplied to the respective ceiling beams. There is one measurement point in a reference room for every circuit. The setpoint of the control valve is therefore the control variable for the ceiling radiant heating system in each circuit.

4.2. Description of the model

The ceiling radiant heating system was modeled by a discrete-time linear time invariant stochastic model. We can consider this model as a Kalman filter giving an estimate of $\hat{x}(k)$ and $\hat{y}(k)$. Outside temperature prediction and heating water temperature were used as the model inputs. Prediction of outside temperature is composed of two values $T_{\max}$ and $T_{\min}$ defining a confidence interval. The outputs of the model are estimates of inside temperature $\hat{T}_{in}$ and of return water[4] $\hat{T}_{rw}$. This can be formalized according to (21) as

$$\hat{x}(k+1) = A\hat{x}(k) + B\begin{bmatrix} T_{\min}(k) \\ T_{\max}(k) \\ T_{hw}(k) \end{bmatrix} + K(y(k) - C\hat{x}(k))$$
$$\begin{bmatrix} \hat{T}_{in}(k) \\ \hat{T}_{rw}(k) \end{bmatrix} = C\hat{x}(k) + D\begin{bmatrix} T_{\min}(k) \\ T_{\max}(k) \\ T_{hw}(k) \end{bmatrix}, \tag{29}$$

where T_{hw} is temperature of the heating water and T_{in} denotes the inside temperature. System matrices A, B, C and D are to be identified using subspace methods as was described in Section 3.3.2. The state $\hat{x}(k)$ has no physical interpretation, when identified by means of the subspace identification. System order is determined by the identification algorithm. Modeling of the heating system of the CTU building is discussed in detail in Ref. [32].

4.3. Results

We have employed two methods of estimating the savings achieved on the building, based on comparison with a finely tuned weather-compensated controller (which also took weather forecast into account).

The first one was a cross-comparison of energy consumption in particular building blocks based on the difference between the heating and return water temperatures (this is directly proportional to the heat consumption provided that the pumps have a constant flow). In the period from mid-February to the end of the heating season (end of March), the overall savings reached 17–24%, depending on the particular building block.

The second method was based on comparison of calorimeter measurements for the entire building for MPC and said weather-compensated control. The measurements were normalized by outside temperatures and ambient temperature set-points to achieve reliable results. For said period of measurement, MPC achieved 29% savings according to this method.

It should be noted that the heating and return water temperature is being measured by standard industrial thermometers, which suffer from measurement errors, such as noise or offset. This introduces some uncertainty into the results. On the other hand, the calorimeters are installed by the heat provider, so we expect them to be well calibrated (or, at least, they do not measure less than the actual heat); heat payments are also based on the calorimeters. So in the terms of finances, the money savings of were also 29% (there is a flat rate on heat for the building).

Measurement of thermal comfort is always difficult and highly individual. As there are some 1500 employees and 8000 students in the building and there are always some people who complain about the ambient temperature, we decided to take the number of complains as the thermal comfort measure. To achieve objective results, the building occupants were not told about the new heating strategy. Under such conditions, the change in the number of complains was insignificant during the test period.

The results are depicted in Fig. 5. The upper part shows outside temperature, whilst the lower compares reference tracking for weather-compensated and predictive controllers. It can be seen, that the predictive controller heats in advance in order to perform optimal reference tracking, that is, inside comfort, and minimum energy consumption. Two last subfigures compare the efficiency

[3] Open source scientific software package for numerical computations (http://www.scilab.org/).

[4] It is crucial to model return water as an output because it gives a significant information about energy accumulated in the building, moreover it represents the interconnection between heating water and room temperature. Omitting the return water would lead to significant lost of information.

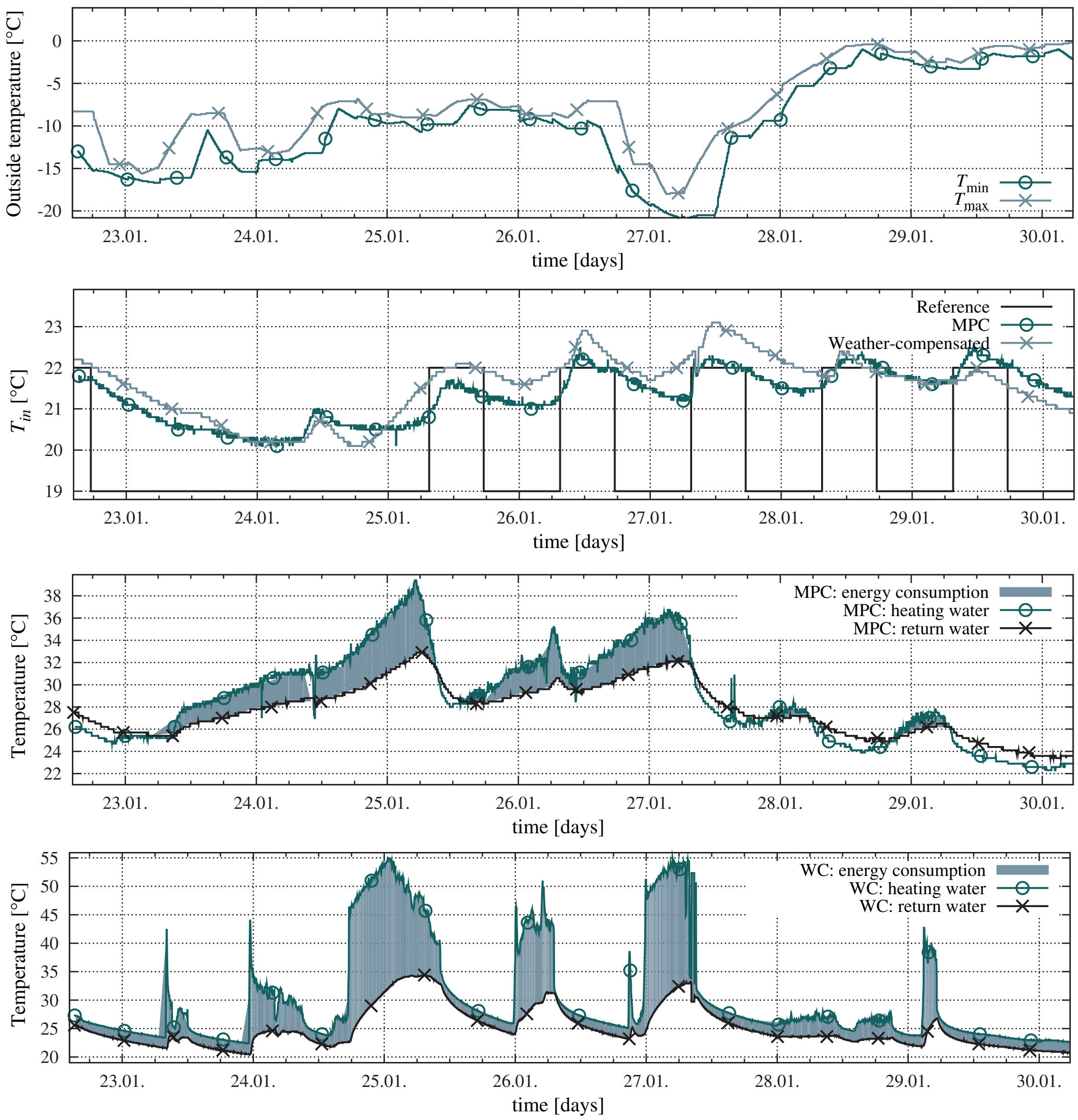

Fig. 5. Different control strategies: comparison of weather-compensated (WC) and predictive control (MPC) of heating water temperature and the room temperature controlled by MPC.

of control measured by energy consumption. The efficiency of the predictive control was superior to the weather-compensated controller, even if the active heating was necessary.

As mentioned before, the building has up to 12 hour heating delay. During weekends, the building cools down and classical heating has to be launched approximately one day before Monday 8 am, depending on the outside temperature.

5. Remarks to future development

Subspace identification methods represent black-box approach to the system modeling. This, alongside with its advantages carries also some drawbacks:

- The system might not be excited enough [22], i.e. the input of the system does not excite the system on satisfactory number of frequencies, thus identification algorithms lack considerable amount of information.
- User may have knowledge of some key feature or characteristics of the physical essence of the system, which is “lost” in the number of data.
- Natural character of the data might pose considerable statistical problem.

One of the most important aspects of the identification is the persistency of the excitation or the excitation itself. Data gathered from the measurement lack some important physical characteristics of the building. One of the possible approaches how to deal with this weak point is generation of artificial data that already contains desired properties. There is also another possibility, more expensive though—specially proposed experiment. It was decided to perform an experiment on real building in through late December 2009 and early January 2010. The comparison of model identification results is depicted in Fig. 6.

It is obvious, that experimental data significantly improved the identification fit. Yet another approach (and much cheaper) how to

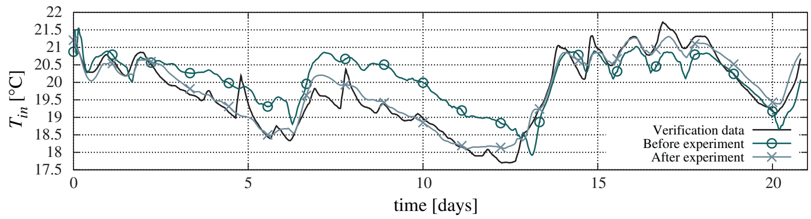

Fig. 6. Validation of model identified from data before and after experiment.

deal with lack of data quality is prior information and its incorporation to the subspace algorithm. Current methods [33] proposed algorithm how to incorporate PI into the algorithm using Bayesian framework. This algorithm makes use of Structured Weighted Lower Rank Approximation (SWLRA) [34] to decompose the projection matrix in order to save special structure, thus keep PI. However, this approach is able to deal with single input single output (SISO) and single input multiple output (MISO) systems only.

Future development of the identification algorithm will try to remedy the above-mentioned problems. Speaking generally, there several approaches to this problem:

- *Bayesian framework*. This approach requires extension to SWLRA algorithm to effectively solve MIMO systems.
- *Incorporation of PI into subspace algorithm*. This approach requires such an computation in subspace identification procedure which enables direct incorporation of PI into system matrices. This approach is the topic of ongoing research.
- *Spectral identification methods*. In robust control, analysis in frequency domain is very popular. The prior information could be incorporated by means of user-defined “filters”. This methodology is also topic of current research.
- *Artificial data*. Generation of data with desired properties is yet another approach. The user incorporates required properties and the knowledge of the physical essence into artificial data which are then used for regular identification. This approach, however, does not explicitly say, how to choose the ratio between artificial and measured data and, therefore, it is only of experimental nature.

In this paper, we treated only predictions of outside temperature because it has dominant influence out of all disturbances affecting the inside temperature. There are, however, other energy sources (like sun intensity, occupancy of the building, etc.). Taking them into account would provide better MPC performance as well as further savings.

6. Conclusion

Predictive control proved to have a great potential in the area of building heating control. The results from real operation on a large university building are very promissing and proved the supremacy of predictive controller over a well tuned weather-compensated control, with the savings of 17–24%. The MPC implementation discussed in the present paper is able to track the desired temperature very accurately, thus maintaining the heating comfort of the building.

However, the MPC strategy requires some extra effort. The crucial part of the controller is the mathematical model of the building. This is not possible by traditional system identification techniques based on statistical identification, as the building data usually do not have the desired statistical properties. On the other hand, finding first principle models is time consuming and not suitable for commercial application. We have shown that a proper identification experiment can provide data suitable for statistical identification, with the help of certain modifications of the standard identification algorithms. Numerical issues of the identification process must be treated very carefully, especially for large-scale systems.

Fortunately, once an appropriate model is found, the MPC tuning is very intuitive and desired properties of the control system can be achieved in a short term. The energy peaks are reduced and the controller does not make fast changes to the control input of the system, which also saves the lifetime of the equipment and reduces the peak energy demands. If desired, it also enables to take different energy prices into account by introducing time-variable tuning parameters into the optimization criterion.

Finally, the decision whether to implement the MPC or not depends largely on the return time of the investments. Even though this largely depends on air temperatures and sunshine during the heating season, the return time for our building is estimated to 2 years. As the identification effort does not really depend on the size of the building, this time will be shorter for large buildings with expensive heating and longer for small buildings with cheap heating.

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