Applied Energy 88 (2011) 3079–3087

Contents lists available at ScienceDirect

Applied Energy

journal homepage: www.elsevier.com/locate/apenergy

Experimental analysis of model predictive control for an energy efficient building heating system

Jan Široký [a,*], Frauke Oldewurtel [b], Jiří Cigler [c], Samuel Prívara [c]

[a] Department of Cybernetics, Faculty of Applied Sciences, University of West Bohemia in Pilsen, Czech Republic
[b] Automatic Control Laboratory, Department of Electrical Engineering, Swiss Federal Institute of Technology in Zurich (ETHZ), Switzerland
[c] Department of Control Engineering, Faculty of Electrical Engineering, Czech Technical University in Prague, Czech Republic

ARTICLE INFO

Article history:
Received 17 November 2010
Received in revised form 15 January 2011
Accepted 8 March 2011
Available online 27 March 2011

Keywords:
Building heating system
Energy savings
Model predictive control
Optimization
System identification

ABSTRACT

Low energy buildings have attracted lots of attention in recent years. Most of the research is focused on the building construction or alternative energy sources. In contrary, this paper presents a general methodology of minimizing energy consumption using current energy sources and minimal retrofitting, but instead making use of advanced control techniques. We focus on the analysis of energy savings that can be achieved in a building heating system by applying model predictive control (MPC) and using weather predictions. The basic formulation of MPC is described with emphasis on the building control application and tested in a two months experiment performed on a real building in Prague, Czech Republic.

1. Introduction

Buildings account for 20–40% of the total final energy consumption and its amount has been increasing at a rate 0.5–5% *per annum* in developed countries [1]. Thanks to developments in the field of mechanical and civil engineering, building energy demands can be reduced significantly. Unfortunately, most of the conventional energy reduction solutions require considerable additional investments. In contrast, energy savings with minimal additional cost can be achieved by improvement of building automation system (BAS). In today's buildings not only heating, ventilation and air conditioning (HVAC) systems can be automatically controlled but also blind positioning and lighting systems can be operated by the BAS [2,3].

The paper focuses on methods that are based on the formulation of the building control as an optimization problem. The building physics are formulated in a mathematical model that is used for the prediction of the future building behavior according to the selected operation strategy and the weather and occupancy forecasts. The aim is to design a control strategy, that minimizes the energy consumption (or operational costs) while guaranteeing that all comfort requirements are met. An advanced control technique usually denoted as Model Predictive Control (MPC) is described in the paper.

A comprehensive overview of the literature related to predictive building control can be found on the web site of the OptiControl project[1]. The key principle of MPC used for building control is the efficient use of the thermal mass or thermal storage of a building. A study presented in [4] was among the first papers which formulated the control of the thermal storage as an optimization problem. The control of a simple solar domestic hot water system considering the weather forecast and two energy rates are discussed there. Some early papers [5,6] deal with a least-cost cooling strategy using the building mass as a thermal storage. An overview of the active use of thermal building mass is given in [7], where a variable energy price and the cost of the peak power are considered in the formulation of the optimization problem. The controller that minimizes cooling costs with respect to the time-varying electrical energy price is presented also in [8]. The aim is to take advantage of night-time electricity rates and to lower the ambient temperature while precooling the chilled water tank. Experimental results of precooling are presented in [9] where a more detailed building load model was used. Predictive control of heating using the thermal mass is discussed in, e.g. [10,11]. Energy savings making use of MPC in relation to different thermal comfort criteria is discussed in [12].

* Corresponding author. Address: Department of Cybernetics, Faculty of Applied Sciences, University of West Bohemia in Pilsen, Univerzitní 8, 306 14 Pilsen, Czech Republic. Tel.: +420 724 030 150.

E-mail addresses: jan.siroky@rcware.eu (J. Široký), oldewurtel@control.ee.ethz.ch (F. Oldewurtel), jiri.cigler@fel.cvut.cz (J. Cigler), samuel.privara@fel.cvut.cz (S. Prívara).

[1] www.opticontrol.ethz.ch.

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doi:10.1016/j.apenergy.2011.03.009

Besides the energy minimization, predictive control can also contribute to energy peak reductions [13,14]. Energy peak reduction can significantly lower the costs of the building operation and initial cost of mechanical parts if considered in the building design. Current grid load and energy peak reduction was considered in [15]. Predictive control used for the sizing of heating systems for discontinuously occupied buildings is discussed in [16], where the model is decoupled into four simple RC models which enable modeling of the contribution of outdoor air temperature, solar radiation, and internal gains separately.

As mentioned, MPC is not the only technique that can be used for optimal building control. There were numerous attempts to utilize advanced control techniques that are well-known in industrial process control also for building control [17]. We briefly mention some of recently published alternative solutions to optimal building control. The general dynamic programming problem for the control of a borehole thermal energy storage system is solved in [18]. The aim was to guarantee the delivery of heat or cold all-year-around while minimizing the operational costs. A reinforcement learning technique used for a building thermal storage control is outlined in [19,20]. The real building experiment provided only 8.3% cost savings because the thermal storage has been only partially utilized by the learning control strategy. In [21], a set of fuzzy rules was used to cut down the time needed for tuning the supervisory controller. Genetic algorithms and simulated annealing were used for optimal control of cooling in [22]. The objective was to design economically optimal the use of natural ventilation, fan-driven ventilation, and mechanical air conditioning with respect to indoor temperature requirements. The unmanageable number of possible control sequences is reduced by consideration of practical issues based on physical insight.

The increased popularity of MPC usage for building control in recent years is indisputable, however, most of the results are based on the simulations or short time experiments. In this paper, we provide a detailed description of an MPC implementation on a real building and we analyze results from two months of operation. The paper is organized as follows. The predictive control strategy is presented in Section 2. Section 3 is devoted to modeling with stress on statistical modeling. A detailed case-study is discussed in Section 4. The Section 5 concludes the paper.

2. Model predictive control

The Building Automation System (BAS) aims at controlling heating, cooling, ventilation, blind positioning, and electric lighting, of a building such that the temperature, CO_2 and luminance levels in rooms or building zones stay within the desired comfort ranges. One typically divides the control hierarchy into two levels: the low-level controller which typically operates at the room-level and is used to track a specified setpoint, and a high-level controller which is done for the whole building and determines the setpoints for the low-level controllers. The article focuses on the usage of Model Predictive Control (MPC), which is used as high-level controller.

2.1. MPC strategy

MPC is a method for constrained control which originated in the late seventies and early eighties in the process industries (oil refineries, chemical plants, etc.) (see, e.g. [23–26]). MPC is not a single strategy, but a class of control methods with the model of the process explicitly expressed in order to obtain a control signal by minimizing an objective function subject to some constraints. In building control one would aim at optimizing the energy use or cost subject to comfort constraints.

During each sampling interval, a finite-horizon optimal control problem is formulated and solved over a finite future window. The result is a trajectory of inputs and states into the future satisfying the dynamics and constraints of the building while optimizing some given criteria. In terms of building control, this means that at the current point in time, a heating/cooling, etc. plan is formulated for the next several hours to days, based on predictions of the upcoming weather conditions. Predictions of any other disturbances (e.g., internal gains), time-dependencies of the control costs (e.g., dynamic electricity prices), or of the constraints (e.g., thermal comfort range) can be readily included in the optimization.

The first step of the control plan is applied to the building, setting all the heating, cooling and ventilation elements, then the process moves one step forward and the procedure is repeated at the next time instant. This receding horizon approach is what introduces feedback into the system, since the new optimal control problem solved at the beginning of the next time interval will be a function of the new state at that point in time and hence of any disturbances that have acted on the building.

Fig. 1 summarizes the basic MPC control scheme. As time-varying design parameters, the energy price, the comfort criteria, as well as predictions of the weather and occupancy are input to the MPC controller. One can see that the modeling and design effort consist of specifying a dynamic model of the building, as well as constraints of the control problem and a cost function that encapsulates the desired behavior. In each sampling interval, these components are combined and converted into an optimization problem depending on the MPC framework chosen. A generic framework is given by the following finite-horizon optimization problem:

Problem 1.

$$\min_{u_0,\dots,u_{N-1}} \quad \sum_{k=0}^{N-1} l_k(x_k,u_k) \qquad \text{Cost function} \tag{1}$$

s.t.

$$x_0 = x \quad \text{Current state} \tag{2}$$

$$x_{k+1} = f(x_k,u_k) \quad \text{Dynamics} \tag{3}$$

$$(x_k,u_k) \in \mathscr{X}_k \times \mathscr{U}_k \quad \text{Constraints} \tag{4}$$

where $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^m$ is the control input, k is the time step, $\mathscr{X}_k$ and $\mathscr{U}_k$ denote the constraints sets of the state and inputs respectively and are explained below. We now detail each of the four components in the above MPC formulation and discuss how they affect the system and the resulting optimization problem. Please note that this is not a comprehensive overview of MPC formulations, but rather a collection of formulations, which are frequently used or reasonable in the field of building control. For a more comprehensive overview on MPC formulations, the reader is referred, e.g. to [27].

2.1.1. Cost function

The cost function generally serves two purposes:

- **Stability.** It is common to choose the structure of the cost function such that the optimal cost forms a Lyapunov function for the closed loop system, and hence will guarantee stability. In practice, this requirement is generally relaxed for stable systems with slow dynamics, such as buildings, which leaves the designer free to select the cost strictly on a performance basis.
- **Performance target.** The cost is generally, but not always, used to specify a preference for one behavior over another, e.g., minimum energy or maximum comfort.

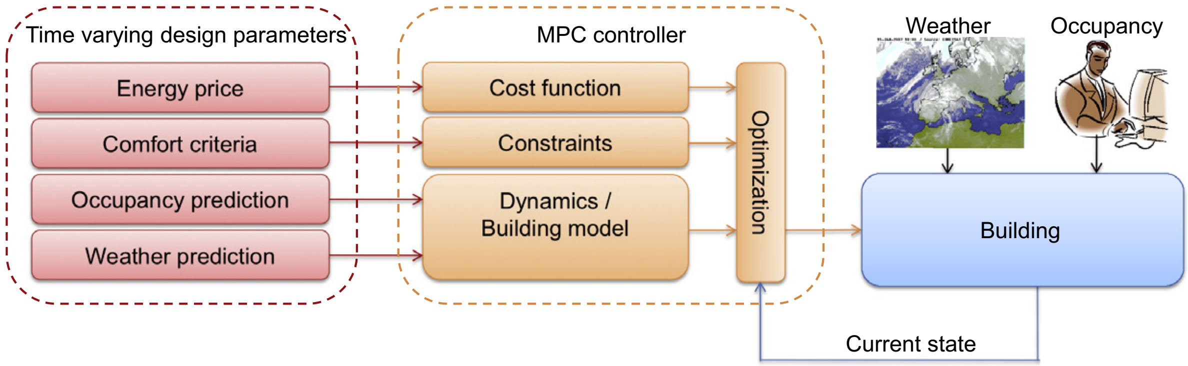

Fig. 1. Basic principle of model predictive control for buildings.

Generally, the main goal is to minimize energy cost while respecting comfort constraints, which can be formalized by the following cost function:

$$l_k(x_k, u_k) = (y_k - y_{r,k})^T Q_k (y_k - y_{r,k}) + R_k u_k, \tag{5}$$

where Q_k and R_k are time-varying matrices of appropriate size and $y_{r,k}$ the reference signal at time k. The trade-off between precision of reference tracking and energy consumption is expressed by proportion of the matrices Q_k and R_k. The reference tracking is expressed as a quadratic form because it significantly penalizes larger deviations from the reference. The energy bill is usually an affine function of a total amount of consumed energy. Therefore, the control cost is weighted linearly. The function Eq. (5) is not the only cost function applicable to building control. There could be, for example, peak energy demand penalization included in the energy bill that can be expressed by L^∞ norm of control inputs in the cost function. Detailed description of the cost function used in the Prague building is given in Section 4.3, for alternative formulations see [28].

2.1.2. Current state

The system model is initialized to the measured/estimated current state of the building and all future (control) predictions begin from this initial state x. Depending on what the state of the building is describing, it might not be possible to measure everything directly. In this case, a Kalman filter can be used to estimate the current state of the building and the estimate is used as initial state.

2.1.3. Dynamics

The controller model, i.e. the mathematical description of the building dynamics is a critical piece of the MPC controller. For the work presented in this paper we restrict ourselves to linear dynamics

$$x_{k+1} = Ax_k + Bu_k. \tag{6}$$

This is the most common model type and the only one that will result in a convex and easily solvable optimization problem.

2.1.4. Constraints

The ability to specify constraints in the MPC formulation and to have the optimization routine handle them directly is the key strength of the MPC approach. There can be constraints on the states or the output, as well as on the input. When explaining different forms of constraints in the following we will do it for input constraints only, but everything applies for state and output constraints alike. *Linear constraints* are the most common type of constraint, which are used to place upper/lower bounds on system variables

$$u_{\min,k} \leqslant u_k \leqslant u_{\max,k}, \tag{7}$$

or generally formulated as

$$G_k u_k \leqslant g_k. \tag{8}$$

The constraints can be constant, given by physical or logical limitations. For instance, valve cannot be open more that 100% or temperature of heating water cannot exceed some predefined level. The constraints can be also time-varying, e.g. to account for different comfort constraints during day-time and night-time. In general case, the constraints can be a function of state variables or inputs as discussed in Section 4.3. This class of constraints can also be used to approximate any convex constraint to an arbitrary degree of accuracy. Linear constraints also result in the simplest optimization problems. Furthermore, one might want to constrain the rate of change, which is done by imposing a constraint of the form

$$|u_k - u_{k-1}| \leqslant \Delta_{u_{\max}}. \tag{9}$$

3. Modeling

Modeling of the building requires insight both into control engineering as well as into HVAC engineering. Moreover, it is also the most time demanding part of designing the MPC setup.

Two approaches to building modeling are outlined in this section. Both of them come from so-called RC modeling. The aim is to provide insight into these techniques with emphasis on their applicability for MPC. Largely used computer aided modeling tools (e.g. TRNSYS, EnergyPlus, ESP-r, etc.) are not considered here, as they result in complex models which cannot be readily used for control purposes.

When large measurement data sets are available, a purely statistical approach for creation of a building model is preferred. A large number of System Identification methods exists (a survey is listed in, e.g. [29]), however, only a few of them have the capability of identification of multiple-input multiple-output (MIMO) systems, which are considered in case of building control. For identification of linear MIMO models, subspace identification methods are often used [29–31] and have been suggested for identification of building models as in [32].

Alternatively to the statistical approach, especially if there is a lack of data or some knowledge of building physics is present, the RC modeling can be used.

3.1. RC modeling

The principle of the thermal dynamics modeling can easily be described by a small example as given in Fig. 2. The room can be thought of as a network of first-order systems, where the nodes

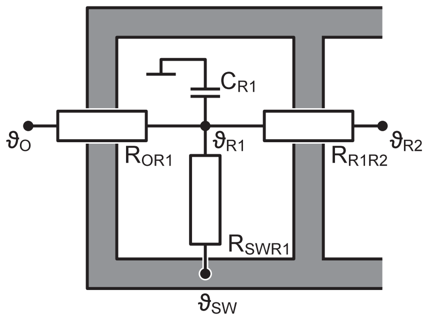

Fig. 2. RC modeling is based on the description of heat transmission between nodes that are representing temperatures. The figure captures example with two rooms where, ϑ_{R1} and ϑ_{R2} are the temperatures in the room $R1$ and $R2$, respectively, ϑ_0 is the outside temperature, ϑ_{SW} is the temperature of the supply water used for floor heating, C_{R1} denotes the thermal capacity of the room $R1$. Resistances are representing the thermal resistances between the nodes.

are the system states and these represent the room temperature or the temperatures in the walls, floor or ceiling. Then the heat transfer rate is given by

$$\frac{dQ}{dt} = K_{ie} \cdot (\vartheta_e - \vartheta_i)$$
$$\Rightarrow \underbrace{\frac{dQ}{d\vartheta_i}}_{C_i} \cdot \frac{d\vartheta_i}{dt} = K_{ie} \cdot (\vartheta_e - \vartheta_i), \tag{10}$$

where t denotes the time, ϑ_i and ϑ_e are the temperatures in nodes i and e, respectively, Q is thermal energy, and C_i denotes the thermal capacitance of node i. The total heat transmission coefficient K_{ie} is computed as

$$\frac{1}{K_{ie}} = \frac{1}{K_i} + \frac{1}{K_e}, \tag{11}$$

where the heat transmission coefficients K_i and K_e depend on the materials of i and e as well as on the cross sectional area of the heat transmission. For each node, i.e. state, such a differential equation as in Eq. (10) is formulated. The actuators are direct inputs to the node, which means that their input is added. The modeling of illumination and CO_2 concentration is omitted here for brevity, for more details on RC modeling see [28].

The model parameters (e.g. K_{ie} or C_i in Eq. (10)) can be determined in two ways: by reading from construction plans or by statistical estimation, which is described in the next sections.

3.1.1. Construction plan

Thermal capacities, resistances and other unknown parameters are determined from the construction plan according to the materials used and their tabular values. Simulations of the acquired model are then required to validate the model accuracy. If the model does not correspond to the measured data, parameter adjustment is necessary.

3.1.2. Statistical estimation

In this approach it is assumed that measurements are corrupted by noise, therefore, the model is extended by a stochastic component. The resulting stochastic differential or difference equations are used for estimation with Maximum Likelihood (ML) or Maximum a Posteriori (MAP) methods to get the desired parameters from a measured data set. Also in this case tabular values of the parameters can be used as initial guess, however, they do not need to be specified as accurately as in the previous case, because they will be updated. Software tools for dealing with statistical estimation are described for example in [33–35], some of them provide functionality to certify the resulting model validity using statistical hypothesis tests.

Following the statistical based estimation procedure is a special case of the ML method and can provide a fast way how to identify a discrete-time model of the continuous-time system $\dot{x}(t) = Ax(t) + Bu(t) + w(t)$ from input/output data where the full state is known (i.e. the state of the system corresponds to the system outputs); $A \in R^{n\times n}$, $B \in R^{n\times m}$, $u(t) \in R^m$ is considered to be the control input while $w(t) \in R^n \sim \mathcal{N}(0, \Sigma)$ is the process noise. The system model can be identified using following statistical procedure:

The first step is discretization of the continuous model as described above with sampling period T_s. Discrete-time model will be then the result from the identification procedure.

$$A_d = e^{AT_s} = I + AT_s + \frac{A^2T_s^2}{2} + \ldots \approx I + AT_s$$
$$B_d = \int_0^{T_s} e^{A\tau} \mathrm{d}\tau B \approx \int_0^{T_s} I \mathrm{d}\tau B = T_s B.$$

The presented discretization (in this case the simplest one – zero-order hold) preserves the structure of the system matrices A and B, so that an element of the discrete-time matrices (say, $a_{d_{i,j}}$) corresponds to the element of the continuous-time matrices at the same position ($a_{i,j}$). Therefore, we can then readily estimate the unknown parameters of the discrete-time model, as will be described below.

The data matrices for identification have the following structure:

$$X_k^{k+N} = (x_k, x_{k+1}, \ldots, x_{k+N})$$
$$U_k^{k+N} = (u_k, u_{k+1}, \ldots, u_{k+N})$$
$$E_k^{k+N} = (e_k, e_{k+1}, \ldots, e_{k+N}),$$

where e_k is white zero mean Gaussian noise with an approximate covariance $T_s^2\Sigma$. The estimation of the parameters θ_i within the system matrices (see Eq. (13)) then can be formulated into the least-squares framework as follows:

$$X_1^N = A_d X_0^{N-1} + B_d U_0^{N-1} + E_0^{N-1} = [A_d \ B_d] \begin{bmatrix} X_0^{N-1} \\ U_0^{N-1} \end{bmatrix} + E_0^{N-1}$$

$$\mathrm{vec}\ X_1^N = \left(\begin{bmatrix} X_0^{N-1} \\ U_0^{N-1} \end{bmatrix} \otimes I_{n\times n} \right)^T \mathrm{vec}\ [A_d \ B_d] + \mathrm{vec}\ E_0^{N-1},$$

where (vec •) is vectorization of a matrix and (• ⊗ •) is a Kronecker product of two matrices. In this equation structure, we can add extra lines into the regressors matrix as well as the left-hand side vector for the structure of the matrices A and B to be preserved. Then, the unknown parameters are estimated using weighted least-squares with higher weights on the rows with constraints of the matrices structure.

4. Case study

The presented MPC scheme of Problem 1 was applied to the building heating system of the Czech Technical University (CTU) in Prague, see Fig. 3. MPC was applied there from January 2010 and was operational until the end of heating season in mid-March 2010.

4.1. Description of the building

As can be seen from Fig. 3, the CTU building is composed of four five-floor blocks, three eight-floors blocks and four-level intermediary parts among the respective blocks. All the blocks have the

(a) Sketch of the building. The insulated part comprises the building blocks B_1 and B_2, while the block B_3 is in non-insulated part of the building.

(b) The highest block in front is B_1, on the right from block B_1 are blocks B_2 and B_3.

Fig. 3. The building of CTU in Prague.

same construction and way of use. This provides us with the unique opportunity to compare different control techniques under the same weather conditions, since we can use different controllers in different blocks at the same time. The south part of the building was insulated two years ago and therefore we can evaluate effectiveness of MPC depending on the insulation level as well.

The CTU building uses a Crittall [36] type ceiling radiant heating and cooling system. In this system, the heating (or cooling) beams are embedded into the concrete ceiling that enables the utilization of the thermal capacity of the building. The heating system scheme of one building block is depicted in Fig. 4. The required temperature of supply water is achieved by mixing hot water from a heat exchanger with return water in a three point valve. The three point valve is operated by a low-level controller that maintains the supply water temperature at the setpoint determined by the high-level controller. In case of the CTU building, a PID controller was used as a low-level controller. For each heating circuit, there is one reference room temperature measurement. A detailed description of the heating system is given in [37].

4.2. Modeling of the building block

We consider a building with two heating circuits and two reference rooms, each related to one circuit as depicted above. The differential equations describing the system are as follows:

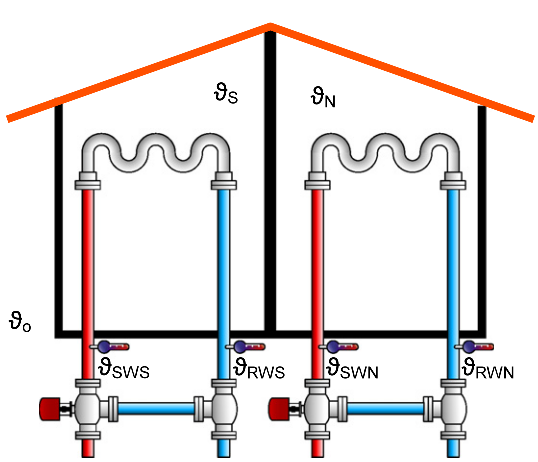

Fig. 4. Simplified scheme of the ceiling radiant heating system.

$$
\begin{aligned}
-\dot{\vartheta}_n &= \frac{1}{C_r R_w}(\vartheta_n - \vartheta_o) + \frac{1}{C_r R_r}(\vartheta_n - \vartheta_s) + \frac{1}{C_r R_{rwr}}(\vartheta_n - \vartheta_{rwn}) \\
-\dot{\vartheta}_s &= \frac{1}{C_r R_w}(\vartheta_s - \vartheta_o) + \frac{1}{C_r R_r}(\vartheta_s - \vartheta_n) + \frac{1}{C_r R_{rwr}}(\vartheta_s - \vartheta_{rws}) \\
-\dot{\vartheta}_{rwn} &= \frac{1}{C_{rw} R_{rwr}}(\vartheta_{rwn} - \vartheta_n) + \frac{1}{C_{rw} R_{rw}}(\vartheta_{rwn} - \vartheta_{swn}) \\
-\dot{\vartheta}_{rws} &= \frac{1}{C_{rw} R_{rwr}}(\vartheta_{rws} - \vartheta_s) + \frac{1}{C_{rw} R_{rw}}(\vartheta_{rws} - \vartheta_{sws})
\end{aligned}
\tag{12}
$$

The meaning of the variables and coefficients is explained in Table 1.

Considering the system state as $x^T = [\vartheta_s \vartheta_{rws} \vartheta_n \vartheta_{rwn}]^T$ and the input vector as $u^T = [\vartheta_o \vartheta_{sws} \vartheta_{swn}]^T$, the state-space model can be formulated in the following way:

$$
\begin{aligned}
\dot{x} = &\begin{bmatrix} -\frac{1}{C_r R_w} - \frac{1}{C_r R_r} - \frac{1}{C_r R_{rwr}} & \frac{1}{C_r R_{rwr}} & \frac{1}{C_r R_r} & 0 \\ \frac{1}{C_{rw} R_{rwr}} & -\frac{1}{C_{rw} R_{rw}} - \frac{1}{C_{rw} R_{rwr}} & 0 & 0 \\ \frac{1}{C_r R_r} & 0 & -\frac{1}{C_r R_w} - \frac{1}{C_r R_r} - \frac{1}{C_r R_{rwr}} & \frac{1}{C_r R_{rwr}} \\ 0 & 0 & \frac{1}{C_{rw} R_{rwr}} & -\frac{1}{C_{rw} R_{rwr}} - \frac{1}{C_r R_{rw}} \end{bmatrix} x \\
&+ \begin{bmatrix} \frac{1}{C_r R_w} & 0 & 0 \\ 0 & \frac{1}{C_{rw} Rz} & 0 \\ \frac{1}{C_r R_w} & 0 & 0 \\ 0 & 0 & \frac{1}{C_{rw} Rz} \end{bmatrix} u.
\end{aligned}
\tag{13}
$$

Finally, the parameters of this predefined system structure are estimated according to the procedure described in Section 3.1 whereas, in this case, the discrete-time system matrices have the following structure:

Table 1
Notation of the variables and coefficients used in the equations describing a building block.

Notation	Description
R_w	Outside wall heat resistance
R_{rwr}	Return water-to-room transition resistance
R_r	Room-to-room transition resistance
R_{rw}	Return water resistance
C_{rw}	Thermal capacity of return water
C_r	Thermal capacity of room
ϑ_o	Outside temperature (from weather forecast)
ϑ_n	Reference room temperature – north side
ϑ_s	Reference room temperature – south side
ϑ_{rwn}	Return water temperature – north side
ϑ_{rws}	Return water temperature – south side
ϑ_{swn}	Supply water temperature – north side
ϑ_{sws}	Supply water temperature – south side

$$A_d = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & 0 \\ \theta_4 & \theta_5 & 0 & 0 \\ \theta_3 & 0 & \theta_1 & \theta_2 \\ 0 & 0 & \theta_4 & \theta_5 \end{bmatrix}, \qquad B_d = \begin{bmatrix} \theta_6 & 0 & 0 \\ 0 & \theta_7 & 0 \\ \theta_6 & 0 & 0 \\ 0 & 0 & \theta_7 \end{bmatrix}$$

Validation of the identified model was carried out by comparison of open loop simulation with verification data set collected during Christmas 2009 (see Fig. 5). The merit of the proposed identification method can be especially seen in well identified trends of heating-up and cooling down.

4.3. Description of the controller

4.3.1. Control objectives

There are several requirements to be fulfilled:

4.3.1.1. Comfort requirements. The reference trajectory $y_{r,k}$, room temperature in our case, is known a priori, as a schedule. The major advantage of MPC is the ability of computing the outputs and corresponding input signals in advance, that is, it is possible to avoid sudden changes in the control signal and undesired effects of delays in the system response.

The schedule defines two minimal levels of the room temperature – during the day, the desired temperature is 22 °C while at night and during weekends there is a setback to 19 °C. One solution how to deal with minimal temperature requirement is to use reference tracking with dynamic cost which is difficult to tune and does not provide possibility for extension to more than two minimal temperature levels [16]. Another solution is to use it as a constraint. This can lead to infeasible problem in some situations. Moreover, there is a tolerance in proposed comfort criterion and therefore it can be useful to slightly violate comfort requirements if it results in considerable energy reduction. Thus, we proposed an alternative MPC problem formulation – the displacement below the reference trajectory is penalized in the criterion. Note, that the 2-norm was used for the weighting of the tracking error – for more accurate performance.

4.3.1.2. Minimization of energy consumption. As the return water circulates in the heating system (see Fig. 4), the energy consumed by the heating-up of the building is linearly dependent on the positive difference between heating ϑ_{sw} and return water ϑ_{rw} temperatures entering/exiting the three port valve in Fig. 4.

Thus, the 1-norm of weighted inputs is to be minimized.

4.3.2. MPC problem formulation

At first, the given system from Section 4.2 is partitioned as follows:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \\ z_k &= Vx_k + Wu_k, \end{aligned}$$

where y_k stands for outputs with reference signal (e.g. $\vartheta_{in,k}$), whilst z_k represents the input-output differences – in our case $z_k = \vartheta_{sw,k} - \vartheta_{rw,k}$.

The requirements (see Section 4.3.1) for the weighting of the particular variables can be carried out by adding the slack variables a_k and b_k which are of same dimension as y_k and z_k, respectively. The resulting optimization problem can be written as follows:

$$J = \min_{a_k, b_k, u_k} \sum_{k=0}^{N-1} a_k^T Q a_k + R b_k$$

$$\begin{aligned} y_{r,k} - y_k - a_k &\leqslant 0, \quad a_k \geqslant 0 \\ z_k - b_k &\leqslant 0, \quad b_k \geqslant 0 \\ u_{\min} \leqslant u_k &\leqslant u_{\max} \\ |u_k - u_{k-1}| &\leqslant \Delta_{u_{\max}} \end{aligned} \tag{14}$$

$$y_k = CA^k x_0 + \sum_{i=0}^{k-1} CA^{k-i-1} Bu_i + Du_k$$

$$z_k = VA^k x_0 + \sum_{i=0}^{k-1} VA^{k-i-1} Bu_i + Wu_k.$$

Q and R stand for the weighting matrices of appropriate dimensions. The weighting matrices are constant because there is a flat rate for energy and the minimal room temperature defined by $y_{r,k}$ has to be maintained over whole the day with the same importance. Each building block requires different amount of energy for maintaining the same comfort therefore the proportion of the weighting matrices Q and R had to be tuned-up for each block separately. The physical limits of the heating system are expressed by constants $u_{\min}$, $u_{\max}$ and $\Delta_{u_{\max}}$. The lower limit for heating water temperature $u_{\min}$ was set to 20 °C, the upper limit for heating water temperature $u_{\max}$ was set to 55 °C and the maximum rate of change of the input signal $\Delta_{u_{\max}}$ that prevents the heating system from heat shocks was set to 20 °C/20 min. The temperature of supply water is controlled by the three point valve. Therefore, the lower limit $u_{\min}$ is not, in fact, a constant value but it is given as minimum of return water temperature and hot water from the heat exchanger. However, this can be neglected because just lower comfort limit is maintained and delivery of warmer supply water than predicted do not result in comfort criteria violation.

Eq. (14) can be readily rewritten into a quadratic programming (QP) problem and solved using a standard QP solver.

The prediction horizon N was chosen to be two days (the system was sampled with a sampling period of 20 min, i.e. $N = 144$) which is a trade-off between accuracy of the weather prediction and a sufficient length of the prediction horizon.

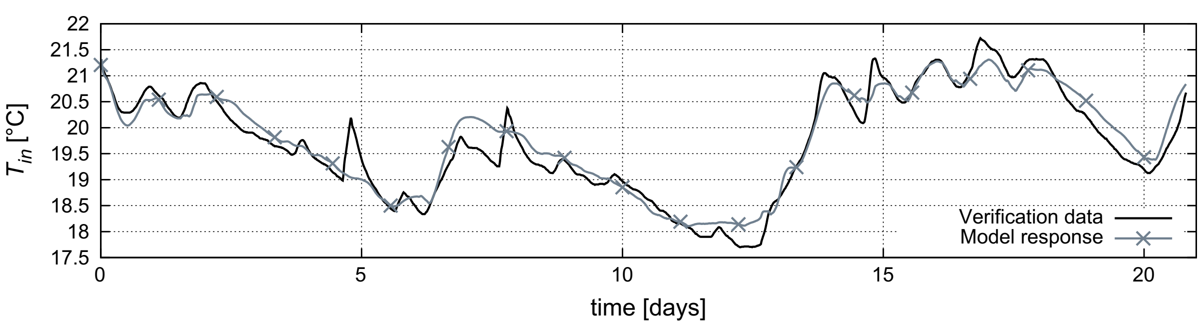

Fig. 5. Validation of model response against verification data set.

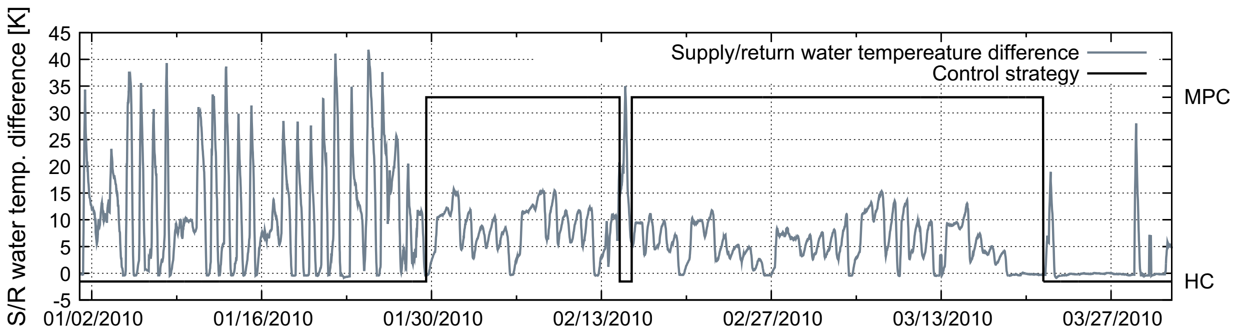

Fig. 6. Heating curve and MPC energy requirements profile.

4.4. Technical setup description

The building was operated by RcWare[2] BAS system. The RcWare system provides data from several weather forecasting servers. In case of the CTU building, weather forecast from National Oceanic and Atmospheric Administration[3] was used. The MPC was implemented in Scilab[4]. The optimization problem was solved by means of Scilab internal linear quadratic programming solver. The computation time was in average 21 s on a PC with Intel Core2 DUO CPU 2.5 GHz. Setpoints for supply water temperature were periodically computed by the following sequence

1. Retrieve the current state ϑ_{swn}, ϑ_{sws}, ϑ_{rwn}, ϑ_{rws}, ϑ_n, ϑ_s from BAS
2. Generate reference room temperature $y_{r,k}$, $k \in 0,\ldots,N-1$ according to BAS setting
3. Download weather forecast $\vartheta_{o,k}$, $k \in 0,\ldots,N-1$
4. Execute MPC scripts in Scilab
5. Apply new setpoints for ϑ_{swn}, ϑ_{sws} into BAS

Because of network communication and interaction between different environments, it was necessary to handle potential failures. In such cases, BAS switched to a backup strategy based on a heating curve and sent a SMS to the operator.

4.5. Investigations setup

Evaluation of the energy savings achieved by different control strategies is a complicated task. The weather conditions change all the time, as well as the number and behavior of the building occupants. Single comparisons of results are affected by these disturbances, therefore two independent comparisons of the real building experiment will be presented.

The first comparison denoted as cross comparison uses almost similar building blocks B_1 and B_2 [5]. The cross comparison had two phases, each lasted for a week. In the first week, block B_1 was controlled by the heating curve and block B_2 by MPC. The other week, the control strategies were switched. The advantage of the cross comparison is compensation of the majority of disturbances because both building blocks are exposed to the same weather conditions.

The second comparison is based on the utilization of so-called heating degree days (HDD) for the normalization of the building energy consumption. HDD is a quantitative index designed to reflect the demand for energy needed to heat a building. There are several methods of HDD computation. In this paper, the outside temperature is subtracted from the required room temperature and this number is summed over the analyzed time period

$$HDD = \sum_{k=T_{begin}}^{T_{end}} y_{r,k} - \vartheta_{o,k}, \qquad (15)$$

where T_{begin}, T_{end} denote the beginning and the end of the measured period, respectively. The method is not precise, especially when outside weather conditions differ a lot. In order to minimize the negative effect of different weather conditions time periods with similar average outside temperature were selected for the comparison.

Because the heating water flow is constant, the sum of difference between the supply water temperature and the return water temperature can be used as energy consumption measure (denoted as E_{CM})

$$E_{CM} = \sum_{k=T_{begin}}^{T_{end}} (\vartheta_{sws,k} - \vartheta_{rws,k}) + (\vartheta_{swn,k} - \vartheta_{rwn,k}). \qquad (16)$$

4.6. Results from real implementation

The Crittall heating system utilizes the building mass as a thermal storage. When the building was operated by a heating curve, the concrete construction was preheated during the night and the heating system was switched off in the morning. The strategy realized by MPC was different; the MPC preheated the concrete mainly at night but it did not switch off the heating during the day. The beneficial side effect of MPC strategy was a significant energy peak reduction as can be seen at Fig. 6. The aim of the energy peak reduction was not explicitly expressed in the problem formulation, it was just a result of the optimization process.

The cross comparison results are summarized in Table 2. According to this comparison, MPC saved approximately 16% of energy in both weeks.

The results from HDD based comparison are in Table 3. It can be seen, that the non-insulated block B_3 required nearly twice as much energy as the insulated blocks B_1 and B_2. The relative savings were more significant at insulated building blocks B_1 (28.74%) and B_2 (26.83%). Nevertheless, at the block $B_3$the relative savings were more than 17% even if there was a significant increase of the room temperature. The absolute MPC savings were larger at the non-insulated block B_3.

The average outside temperature during the cross comparison was −2.3 °C, while during the comparison based on HDD was 3.4 °C. In case of lower outside temperatures, the energy has to be continuously supplied to the building and the active usage of building heat accumulation is limited. This could be the reason why saving estimation based on the cross comparison is lower than savings estimation based on HDD.

[2] http://www.rcware.eu.
[3] http://www.noaa.gov.
[4] http://www.scilab.org.
[5] Block B_1 uses slightly more energy than block B_2, it can be seen in Table 3. This fact was considered in the cross comparison.

Table 2
Comparison of heating curve (HC) and model predictive control (MPC) strategies using similar building blocks B_1 and B_2.

		B_1		B_2		
	Mean ϑ_o (°C)	Control	Mean ϑ_s, ϑ_n (°C)	Control	Mean ϑ_s, ϑ_n (°C)	MPC savings (%)
1st week	−3.4	HC	21.4	MPC	21.1	15.54
2nd week	−1.3	MPC	21.4	HC	20.9	16.94

Table 3
Heating degree days based comparison. The ratio E_{CM}/HDDexpresses normalized energy demands for heating.

Block-control	E_{CM}/HDD	Mean ϑ_o(°C)	Mean ϑ_s, ϑ_n(°C)	Days compared	Relative MPC savings (%)
B_1-HC	0.906	3.8	21.6	84	28.74
B_1-MPC	0.645	3.2	21.8	49	
B_2-HC	0.813	4.0	21.7	85	26.83
B_2-MPC	0.595	3.0	21.7	49	
B_3-HC	1.532	3.8	20.9	84	17.67
B_3-MPC	1.262	3.2	21.9	49	

5. Conclusion

It was shown that the energy savings potential for using MPC with weather predictions for the investigated building heating system were between 15% and 28% depending on various factors, mainly insulation level and outside temperature. This is consistent with results achieved in large scale simulations done in scope of the Opticontrol project ([28] chapter 8). The real building application results are very encouraging, nevertheless, two issues have to be mentioned. First, each building is unique and the MPC saving potential is dependent on many factors as HVAC system, building construction or weather conditions to name a few. Second, the complete cost benefit analysis should not include just energy savings but also the cost of the MPC implementation, i.e. foremost the modeling effort, that presents the most time consuming part and MPC integration into a BAS. In contrast to the current building control techniques, MPC is based on a non trivial mathematical background that complicates its usage in practice. But its contribution to reduction of a building operation cost is so significant that it is expected that it will become a common solution for so-called intelligent buildings in a few years.

Acknowledgment

This work has been supported from the state budget of the Czech Republic, through the Ministry of industry and commerce, in the scope of Grant No. 2A-1TP1/084, *"Integration of building systems, research and application of intelligent algorithms with influence on energy consumption of buildings and living houses"*.

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