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Model Predictive Control of a Geothermally Heated Bridge Deck

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Abstract— **This paper describes the application of model predictive control (MPC) to a geothermal bridge deck heating system. The control system integrates concepts of MPC with a first-principles bridge deck model and hourly computerized National Weather Service (NWS) forecasts to prevent bridge icing without the use of salt or other chemical de-icing materials. The proactive nature of the control system maximizes motorists' safety and bridge life while minimizing system operating costs. Results are presented for a snow event on a ¼-scale, hydronically-heated bridge deck. The effect of MPC error penalty and move suppression weighting on bridge deck heating performance is discussed.**

I. INTRODUCTION

Bridge icing, especially preferential bridge icing (formation of ice on a bridge deck before ice appears on approaching sections of road), represents a major transportation safety issue. A common response is the application of salt to suppress the freezing point and prevent ice formation. Unfortunately, this creates two problems of major concern. The first is the environmental impact associated with salt runoff in water bodies. The second is a reduction in bridge life due to the corrosive effects of salt on rebar and other structural steel. One alternative to avoid salt is the use of heated bridge technology (HBT). This paper describes a control system for this approach.

In 1999, the U.S. Department of Transportation Federal Highway Administration funded the Oklahoma State University (OSU) Geothermal Smart Bridge Project. The mission of this project was to "research, design, and demonstrate technically feasible, economically acceptable, and environmentally compatible Smart Bridge systems to enhance the nation's highway system safety and to reduce its life cycle cost [1]." The OSU project uses a hydronically-heated deck with geothermal energy as the heat source. The control system described in this paper represents a major step forward from earlier approaches used to control bridge deck heating. The philosophy and capabilities of the control system described in this paper are consistent with the goals for Intelligent Transportation Systems.

A hydronically-heated bridge deck has tubes buried in the pavement. Heat is transferred to the bridge deck when a warm fluid is pumped through the tubes. For the application described in this paper, a ground source heat pump, which recovers energy stored in the earth, is used to heat the fluid circulated through the bridge deck. Energy is supplied to the heat pump from a ground loop heat exchanger. The ground loop heat exchanger utilizes a second fluid circulating through tubing buried in the earth. Fig. 1 shows a schematic of the system.

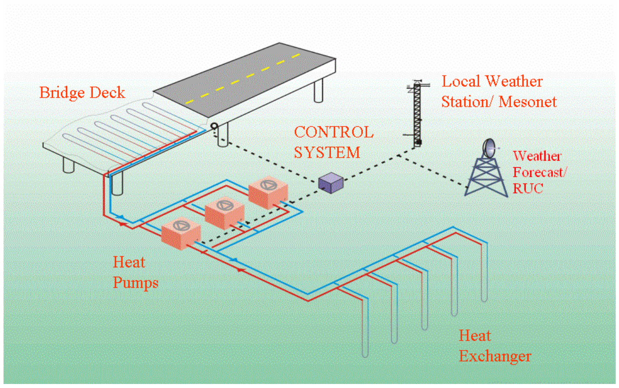

Fig. 1. OSU hydronically-heated bridge deck. To show the advantage of heated bridge technology, only half of the test bridge is equipped with the heating system.

Funding for HBT research was provided between 1992 and 1997 as part of the Applied Research and Technology program (Section 6005) of the Intermodal Surface Transportation Efficiency Act. A total of eight heated bridge decks were constructed in five states as a result of this program. Table I summarizes the control systems used on these bridge decks. The information presented in Table I was derived from a report [2] published in July 1999 by the Office of Bridge Technology. The report gives the scope, operating controls, construction details, costs, and operating experience for each bridge.

The control systems listed in Table I employ traditional feedback control. All have the capability to remove snow and ice from a bridge deck *after* a freezing event is first detected. Because heating a bridge deck is a slow process (requires hours), the purely reactive nature of feedback control results in an unavoidable accumulation of snow or ice until the bridge deck is heated above 0 °C. With one possible exception, none of the control systems listed in Table I have the ability to prevent preferential icing without manual intervention. Feedforward (proactive) control is required to automatically preheat the bridge deck prior to an expected icing event. To do so, the controller needs continuously updated weather forecasts for the bridge site and a model of how the bridge deck responds to changing

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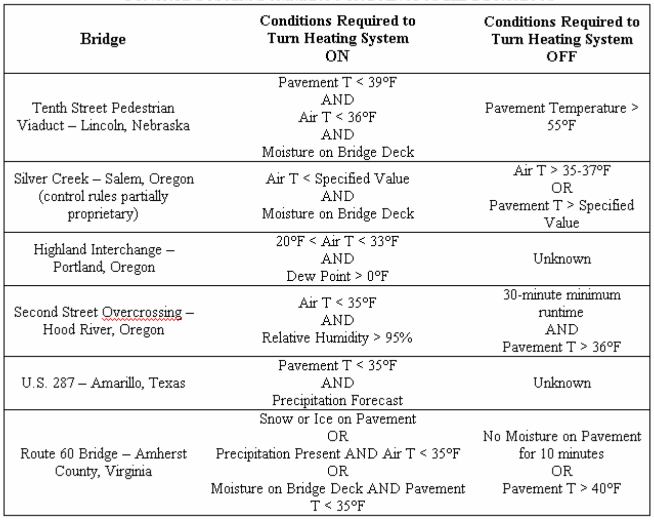
weather conditions. This capability is incorporated in the controller described in the next section.

TABLE I
CONTROL SYSTEM SUMMARY FOR PREVIOUS HBT PROJECTS



Bridge	**Conditions Required to Turn Heating System ON**	**Conditions Required to Turn Heating System OFF**
Tenth Street Pedestrian Viaduct – Lincoln, Nebraska	Pavement T < 39°F AND Air T < 36°F AND Moisture on Bridge Deck	Pavement Temperature > 55°F
Silver Creek – Salem, Oregon (control rules partially proprietary)	Air T < Specified Value AND Moisture on Bridge Deck	Air T > 35-37°F OR Pavement T > Specified Value
Highland Interchange – Portland, Oregon	20°F < Air T < 33°F AND Dew Point > 0°F	Unknown
Second Street Overcrossing – Hood River, Oregon	Air T < 35°F AND Relative Humidity > 95%	30-minute minimum runtime AND Pavement T > 36°F
U.S. 287 – Amarillo, Texas	Pavement T < 35°F AND Precipitation Forecast	Unknown
Route 60 Bridge – Amherst County, Virginia	Snow or Ice on Pavement OR Precipitation Present AND Air T < 35°F OR Moisture on Bridge Deck AND Pavement T < 35°F	No Moisture on Pavement for 10 minutes OR Pavement T > 40°F

II. CONTROL SYSTEM

A. Overview

The ¼-scale OSU test bridge is 6.1 m wide by 18.2 m long (111.5 m^2). The bridge deck was constructed over a man-made pond. A hydronic heating system is used to heat half of the bridge deck (6.1 x 9.1 m). Heat input to the ¼-scale OSU test bridge is provided by fluid (42% propylene glycol and 58% water) circulating through a single geothermal (water-to-water) heat pump. For a full-size bridge, a bank of parallel geothermal heat pumps would be used. The number of heat pumps installed at a heated bridge site would be a function of the size of the bridge and the local weather patterns. A typical bridge design may require eight to ten heat pumps (e.g., for a typical interstate highway in the United States). For an installation with multiple heat pumps, the output of the controller would determine the number of heat pumps that are turned on.

Measurements taken at the bridge site are used to provide feedback control action. Weather forecast inputs in combination with a detailed bridge deck model are used to generate feedforward control action.

The bridge deck temperature is the controlled variable (CV). The CV measurement is an average value provided by an array of thermistors embedded in the bridge deck one-eighth of an inch below the pavement surface.

For the OSU test bridge, the manipulated variable (MV) is the inlet supply temperature of the hot fluid circulating through the bridge deck. This approximates the more general case where the number of heat pumps in use is varied.

The controller [3] generates a new control action every fifteen minutes. The controller uses predicted weather and bridge conditions for the next 12 hours when calculating each control action. The weather variables used as input to the controller include air temperature, sky temperature, precipitation type, precipitation rate, relative humidity, wind speed, wind direction, and solar radiation. Current weather measurements are provided by instruments at the bridge or a nearby site. The weather forecasts are downloaded across the internet from a weather model developed and operated by the National Weather Service (NWS). Other inputs to the controller are the bridge deck and heating system measurements.

B. Motivation to Use MPC

MPC is typically employed for control of constrained, multivariable processes, e.g., petroleum refining. Although SISO in nature, the bridge deck heating problem has several characteristics that make it a candidate for SISO MPC. The most significant are:

1. Poor response dynamics (the bridge deck responds slowly due to high thermal capacitance of the bridge deck)
2. Critical constraints must be satisfied at all times: public safety (bridge deck must remain ice-free); mechanical and rate-of-change constraints associated with the geothermal heat pumps.
3. A first-principles model of a hydronically heated bridge deck is available.
4. Information concerning future weather predictions (forecasts) at the bridge site is available for use by the controller.
5. Operation of the bridge deck heating system needs to be optimized to allow use of the smallest possible heating system (economics are heavily influenced by the additional capital costs of the heating system).

C. Bridge Deck Model

The bridge deck model used in the controller was developed by investigators from the OSU School of Mechanical and Aerospace Engineering [4] – [9]. The bridge deck model uses a system of partial differential equations to describe the energy balance around a hydronically heated bridge deck. A two-dimensional finite difference approach is used to numerically solve this system of equations. The model considers heat transfer due to solar radiation, thermal radiation, convection at the deck surfaces (top and bottom), rain and snow evaporation (sensitive and latent heat effects), conduction through the bridge deck and tube walls, and heat transfer from the bridge loop fluid.

There are four types of inputs to the model: bridge layout parameters, physical property parameters, weather conditions, and heating system parameters. The list of input parameters and variables is provided in Table II. Three outputs are generated by the model: average bridge deck surface temperature, return temperature of the bridge loop fluid, and the heat transfer rate from the bridge loop fluid.

The fidelity of the bridge deck model used in the controller is much better than most employed for MPC in the process industries.

TABLE II
BRIDGE DECK MODEL PARAMETERS

Pavement Length = 9.144 m	Absorptivity Coefficient = 0.6
Pavement Width = 6.096 m	$C_{p,Layer\,1}$ = 2.2x10^6 J/m^3 °C
Slab Orientation = 6° from North	$C_{p,Layer\,2}$ = 0 J/m^3 °C
Pavement Thickness = 0.1524 m	k_{Pipe} = 0.439 W/m °C
Pipe Spacing = 0.3048 m	Wall Thickness of Pipe 1.5875x10^{-3} m
Pipe Diameter = 0.01905 m	Fluid Type 2 = propylene glycol water solution
Pipe Depth Below Surface = 0.0889 m	Weight % GS-4 = 42%
Depth to Interface 1 = 15 m	Number of Flow Circuits = 10
$k_{Layer\,1}$ = 1.618041 W/m °C	Length of Pipe Per Circuit = 19.811 m
$k_{Layer\,2}$ = 0 W/m °C	Transient Time Step = 20 sec
Emmissivity Coefficient = 0.9	Bottom Boundary Condition 1 = convection type
Minimum Flow Condition = 0 kg/sec	

D. Weather Inputs

Current weather conditions were provided by instrumentation installed at the bridge site and a nearby Remote Weather Information System (RWIS) station. The RWIS station is part of the Oklahoma Mesonet [10], [11]. The sample period between weather measurement updates was five minutes.

The nine weather variables used as input to the bridge deck model were listed previously in Section II.A. Two weather input variables, solar radiation and precipitation, stand out in terms of importance, potential measurement difficulty, and/or ability to forecast.

While not difficult to measure, the ability to accurately forecast solar radiation is challenging due to effects of cloud cover. Because the bridge deck model is highly sensitive to solar radiation, the effect of forecast errors on control performance is magnified. Conservative forecast values for solar radiation can be used to guarantee controller performance. The tradeoff is that this approach results in periods of unnecessary bridge deck heating.

From a practical standpoint, accurate measurement of rate of snowfall or freezing precipitation is difficult. As a backup, the control system calculated snowfall and rainfall rates from NWS NEXRAD Doppler radar (WSR-88D) data. Due to high level of uncertainties in calculating ground level precipitation from radar data, actual measurements are strongly preferred.

E. Weather Forecasts

The controller obtained real-time weather forecasts from the NWS Rapid Update Cycle (RUC) model. The RUC is an atmospheric prediction system designed to provide numerical forecast guidance for weather-sensitive users [12]. In 2004, RUC ran at the highest frequency of any of the forecast models at the National Center for Environmental Prediction (NCEP), assimilating recent observations aloft and at the surface to provide frequent updates of current conditions and short-range forecasts using a sophisticated mesoscale model.

RUC provides forecasts for every point in a grid spanning the continental U.S, Canada, and Mexico. Grid points are spaced 20 km apart [13]. Every three hours, beginning at 00:00 GMT, RUC generates 0-hr, 3-hr, 6-hr, 9-hr, and 12-hr forecasts. At the beginning of every hour (GMT) that is not a multiple of three, RUC produces an updated 0-hr and 3-hr forecast [14].

Forecast data acquisition was performed as follows. RUC output data for the entire U.S. was ingested via satellite dish using the National Oceanic and Atmospheric Administration (NOAA)'s port data service. The raw data was then placed on a Linux weather server via a local data manager (LDM). An LDM is a data routing tool that is used in the meteorological community. The weather server ran PERL scripts to extract information for the grid point nearest the bridge site from the raw data files. Extracted weather data was sent via secure FTP to the controller.

The controller computed a control action once every fifteen minutes. For times not coinciding with RUC forecasts, linear interpolation was used. Current measurements at the bridge were used in place of 0-hr RUC forecasts to compensate for RUC forecast errors. At times when RUC provided a three-hour forecast, only the first three hours of the previous forecast vector were replaced.

F. Control System Architecture

The control system was implemented using a three-layer structure (Fig. 2). The top layer is a rule-based meteorological feedforward element. The input to this layer is the RUC forecast. Rules are used to establish whether an icing event is imminent. If the potential for icing exists, the heat pump is turned on and a bridge deck temperature reference trajectory is generated for the MPC controller.

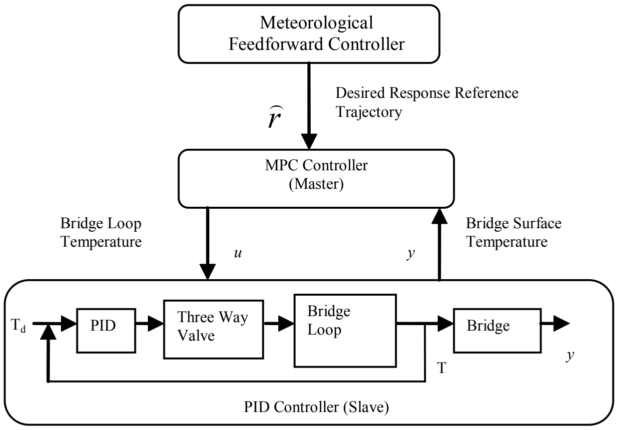


Fig. 2. Three layer control system hierarchy

The middle layer is the MPC controller. It uses the reference trajectory, current bridge surface temperature, weather inputs and forecast, and the first-principle bridge deck model to calculate the set point for a slave PID controller. The PID controller controls the bridge deck inlet supply temperature by diverting a portion of the circulating fluid through a pond-immersed heat exchanger.

MPC was implemented [3], [15] using a modified form of the QDMC algorithm [16]. The prediction and control horizons were 12 hrs and 6 hrs, respectively. The prediction horizon was dictated by the length of the RUC forecast. The controller execution frequency was 0.25 hrs.

The reference trajectory was constructed to maintain the average bridge deck surface temperature above 0 °C when the potential for bridge deck icing exists. For the results shown in the next section, the reference set point was 4 °C. To provide a margin of safety, the reference trajectory was constructed assuming an icing event would occur two hours before first indicated by the RUC forecast. The reference trajectory assumed a maximum preheat ramp rate of 0.5 °C/hr.

III. EXPERIMENTAL RESULTS

This section documents performance of the control system during a snow event recorded on February 4, 2004, between 10:30 and 18:00. Total precipitation was 7.4 mm water equivalent (approximately 74 mm snow). The ambient air temperature recorded at a Mesonet station within a mile of the OSU test bridge is shown in Fig. 3. The arrival of the cold front preceding the snow storm is evident beginning at 10:00 on 2/4/2004.

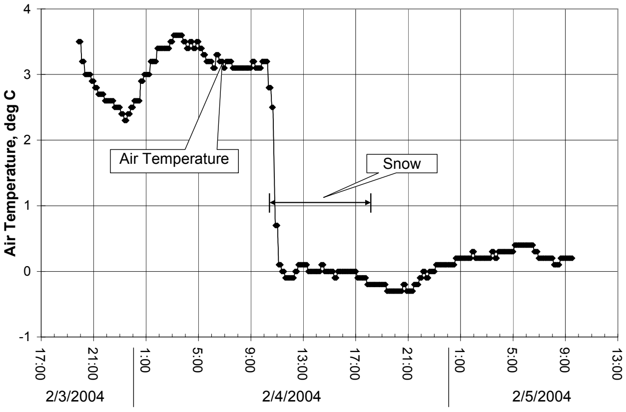


Fig 3. Ambient air temperature at test bridge site.

The controller was configured to achieve an average bridge deck temperature of 4 °C two hours prior the predicted arrival of the cold front. The controller first engaged the heating system at 01:36, nine hours before the snow began. The heat pump was off prior to this time. Fig. 4 shows the bridge deck surface temperature starting to increase around 01:30 on 2/4/2004.

The maximum demand on the heating system occurs during the snow event. The heat of fusion must be provided in order to prevent accumulation of snow on the bridge deck. The control strategy relies on the thermal capacitance of the bridge deck to supplement the heat input when the rate of precipitation exceeds the capacity of the bridge deck heating system. This occurred between 10:30 and 11:00 on 2/4/2004 and is reflected in the decrease in the bridge deck temperature from 4 °C to 1 °C.

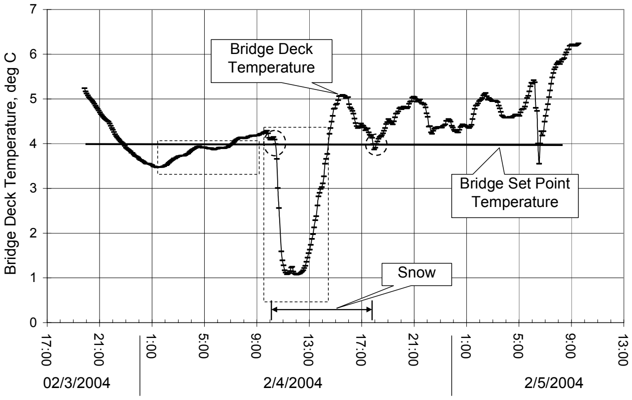


Fig. 4. Bridge deck surface temperature recorded Feb. 3-5, 2004. This is the CV for the bridge deck heating system.

Video records from the webcam installed at the test bridge confirmed that the heated portion of the bridge remained snow-free during the entire event.

The output of the MPC controller is shown in Fig. 5 as the set point to the PID bridge deck supply temperature controller. The recorded inlet supply temperature is shown in Fig. 6.

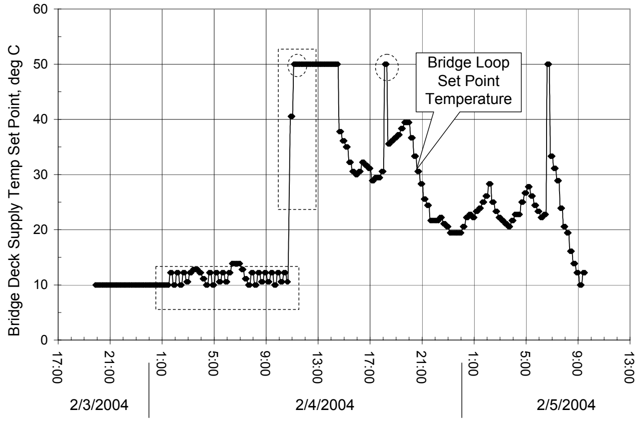


Fig. 5. MPC controller output (set point to slave PID controller used to adjust bridge deck inlet supply temperature).

The MPC controller included 10 °C minimum and 50 °C maximum MV constraints on the bridge deck inlet supply temperature. These constraints were imposed by mechanical considerations for the water-to-water heat pump. The fact that the controller was unable to achieve temperatures above 38 °C during periods when maximum heating was called for (10:45 – 14:30 on 2/4/2004) is a limitation of the system heating capacity.

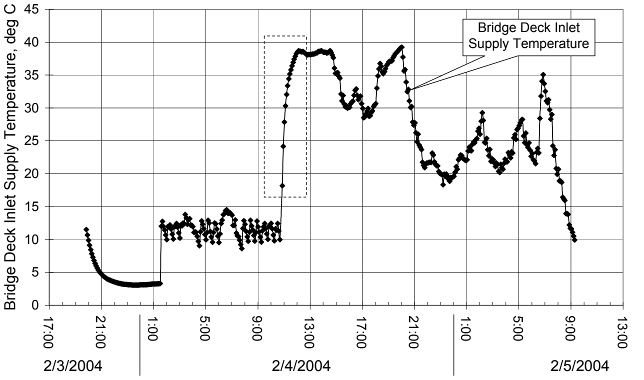


Fig. 6. Bridge deck inlet supply temperature. This is the MV for the bridge deck heating system.

IV. DISCUSSION

A. MPC Controller Tuning

The standard quadratic objective function was used in the MPC algorithm :

$$\min_{\Delta u} \Phi = \left(\hat{e} - A\,\Delta\ \right)\Gamma^{T}\Gamma\left(\hat{e} - A\,\Delta\ \right) + \Delta u^{T}\Lambda^{T}\Lambda\,\Delta u \tag{1}$$

s.t. $u_{min} \le u \le u_{max}$

where: Φ = quadratic objective function

$\hat{e}$ = projected error vector = $\hat{r} - \hat{y}$

$\mathbf{A}$ = dynamic matrix

$\Delta\mathbf{u}$ = sequence of future control actions

$\mathbf{\Gamma}$ = output error weighting matrix

$\mathbf{\Lambda}$ = move suppression weighting matrix

The $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$ matrices are the tuning parameters used to modify performance of the controller. The numeric values used to populate these matrices determine how the controller balances the competing objectives of quick response to errors without excessive control action.

The results in Section III were generated using diagonal matrices divided into three equal intervals, e.g.:

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_{I} & & & & & & & & 0 \\ & \gamma_{I} & & & & & & & \\ & & \gamma_{I} & & & & & & \\ & & & \gamma_{II} & & & & & \\ & & & & \gamma_{II} & & & & \\ & & & & & \gamma_{II} & & & \\ & & & & & & \gamma_{III} & & \\ & & & & & & & \gamma_{III} & \\ 0 & & & & & & & & \gamma_{III} \end{bmatrix} \tag{2}$$

The weighting values for the error penalty matrix $\mathbf{\Gamma}$ were $\gamma_{I} = 2.5, \gamma_{II} = 1, \gamma_{III} = 0.7$. The identity matrix **I** was used for the move suppression matrix, $\mathbf{\Lambda}$. This weighting was used in simulations during initial controller development. After analyzing results from the 2/4/2004 and other snow events, it was determined that this choice of weights caused the bridge deck temperature to track the desired trajectory but with an undesirable lag. The simulation results in Fig. 7 illustrate the observed lag.

A simulation study was performed to identify a better combination of weights. The best reference tracking performance was achieved using $\gamma_{I} = 3, \gamma_{II} = 0.5, \gamma_{III} = 0.1$ and $\lambda_{I} = 0.1, \lambda_{II} = 0.5, \lambda_{III} = 1$. Simulation results for this combination are shown in Fig. 8. Some initial overshoot is observed but this can be tolerated. The ratios of the $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$ elements for this case are [30, 1, 0.1]. The increased emphasis on immediate error reduction is clear from comparison to the ratios of the original weighting [2.5, 1, 0.7].

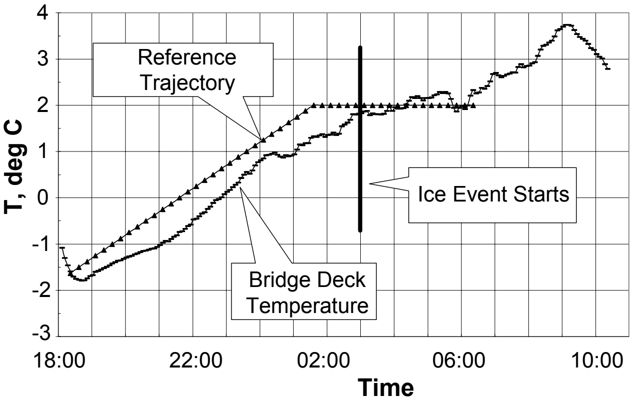


Fig 7: Lag observed with $\mathbf{\Gamma}$ = [2.5, 1, 0.7] and $\mathbf{\Lambda}$ = [1, 1, 1]. The ratios of the $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$ weights are [2.5, 1, 0.7].

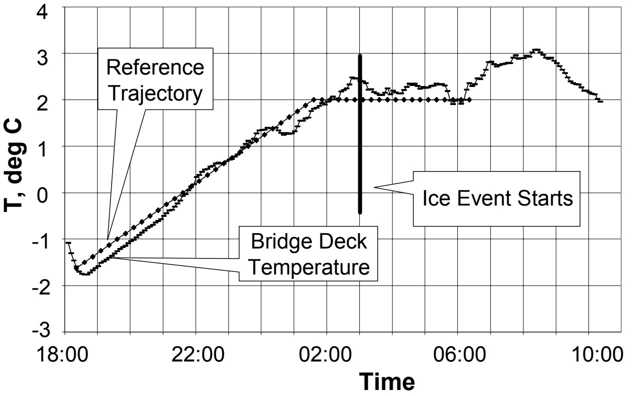


Fig 8: Improved tracking observed with $\mathbf{\Gamma}$ = [3, 0.5, 0.1] and $\mathbf{\Lambda}$ = [0.1, 0.5, 1]. The ratios of the $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$ weights are [30, 1, 0.1].

To be conservative, it would be desirable for the bridge deck temperature to lead the reference trajectory. The following set of weights produced this effect (Fig. 9): $\gamma_{I} = 0.75, \gamma_{II} = 1.25, \gamma_{III} = 1$ and $\lambda_{I} = 0.1, \lambda_{II} = 0.5, \lambda_{III} = 1$. The ratios of the $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$ elements for these weights are [7.5, 2.5, 1]. The fact that these ratios are between the lag (Fig. 7) and no-lag-or-lead (Fig. 8) cases illustrates the complexity of tuning an MPC controller. Additional work is needed to enable prediction of appropriate weights for other bridge configurations and locations.

B. Controller Operation and Maintenance

From a real-time operating standpoint, the most challenging aspect of implementing the MPC controller was creating and maintaining the network connections to provide the weather data and forecasts. Successful implementation could not have been achieved without the participation of a researcher in the College of Atmospheric & Geographic Sciences at the University of Oklahoma. Real-time RUC data was provided through the local data manager (LDM) data stream of the Oklahoma Climatological Survey. The controller logic had to be robust to satellite and network disruptions. Much of the controller development work on the project was directed to this end.

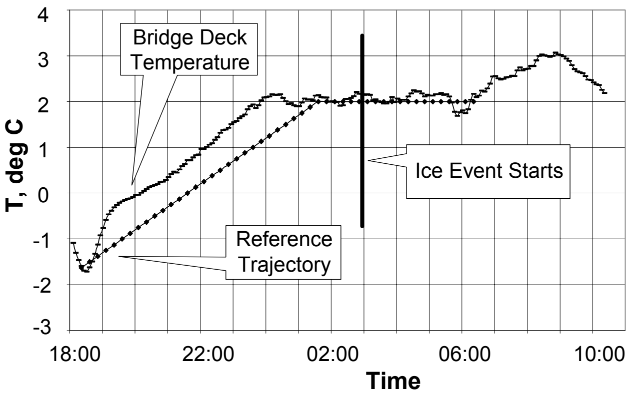


Fig 9: Tracking observed with $\Gamma = [\ 0.75, 1.25, 1]$ and $\Lambda = [0.1, 0.5, 1]$. The ratios of the Γ and Λ weights are [7.5, 2.5, 1].

For practical purposes, the control system for the bridge deck heating system must be robust and simple to operate. The end user will be Department of Transportation personnel. The initial decision to investigate MPC was motivated by the ability to optimize performance and minimize the size and cost of the bridge deck heating system. While the desired controller performance was achieved, excess complexity was required to provide the necessary reliability. Future work on control systems for bridge deck heating systems needs to consider other ways to capture the performance benefits of MPC in a simpler framework.

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